2.0 NOAA POLAR SATELLITE NAVIGATION AND EARTH LOCATION

In this document, when reference is made to satellite navigation, the emphasis is on information representing the satellite orbital position, velocity and orientation. Our goal is to present information that will enhance the user's ability to receive and process polar instrument data.

2.1 NAVIGATING THE POLAR SATELLITE

Any object in orbit about a more massive body will follow Kepler's three laws of motion:

1. The path of the object will be an ellipse, with the massive body at one focus.

2. A straight line joining the central body and the orbiting body will sweep out equal areas in equal times.

3. The square of the sidereal (relative to the stars) period of the orbiting body is directly proportional to the cube of the semi-major axis of the orbit.

These relations are true for any orbit; a planet orbiting a star, a moon orbiting a planet, or an artificial satellite orbiting the Earth. Discussion here will be confined to the last case.

If the Earth was a perfect sphere and there were no nearby bodies, the orbit into which the satellite was placed initially would remain unchanged. However, the presence of the Sun and Moon will cause the orbit to vary. Since the orbits which are being considered here are close to the Earth, the most important cause of variation in the orbit is the non-sphericity of the Earth; the flattening of the poles and the bulging of the equator resulting from the adjustment of the Earth to its rotation. The equatorial bulge is small, the equatorial radius exceeding the polar radius by only about a third of a percent. Nevertheless, this is sufficient to have a major effect on the behavior of a satellite orbit.

Since a satellite is a rotating body, a torque applied perpendicular to the axis of rotation will result in a precession of the rotation axis. Such a torque is supplied by the attraction of the equatorial bulges. The amount of torque, and hence the amount of precession, is governed by two quantities: the mean distance of the satellite from the Earth center, (a - the semi-major axis of the orbit ellipse), and the inclination of the orbital plane to the Earth's equatorial plane, I, measured through 180 degrees from the east direction (the direction of the Earth's rotation).

This situation permits certain characteristics of the orbit to be controlled. The satellite height is usually specified initially in order to achieve the desired Earth coverage with particular instruments. For NOAA satellites, the inclination is then chosen such that the orbit will precess in the same direction and at the same rate as the Earth revolves about the Sun. This situation is termed Sun synchronous, and means that the satellite will preserve its angular relationship with the Sun over time. This causes the satellite to view each latitude at the same Local Solar Time (LST) on each orbit.

The NOAA series of satellites have been placed in orbits with a mean height of about 850 kilometers (semi-major axis about 7228 kilometers). In order to be Sun synchronous, the inclination must then be about 99 degrees. This means that the satellite moves in a westward direction, termed a retrograde orbit, since the satellite motion is opposite to the Earth rotation direction. Since, by Kepler's third law, the period is related to the semi-major axis, the period must be about 102 minutes. The period usually specified for NOAA series satellites is the *nodal* period - the time from one ascending node to the next. Since the orbit is precessing, this will be slightly different from the sidereal period - relative to a fixed point on the celestial sphere. A third period which is often given is the anomalistic period - the time from perigee to perigee. Perigee is the point in the orbit which is closest to the Earth center; this also changes with time.

The LST of the ascending node is not constrained by any of these considerations, and is chosen for other reasons, mainly coverage. Power and thermal constraints preclude normal operation within two hours of noon LST. The times usually chosen are either an ascending (northbound) node around 1430 LST or a descending (southbound) node around 0730 LST.

Thus far only the characteristics of the orbit in space have been considered, with no reference to the physical Earth around which the satellite travels (except for the effects of the equatorial bulges). Although the orbital plane is precessing with a period of one year, the Earth is rotating beneath it once per day, causing the satellite to pass over different geographical areas on each pass. The movement is just the amount that the Earth has rotated in one orbital period, and is 25.5 degrees of longitude (in 102 minutes). Since the Earth rotates from west to east, this displacement is westward.

2.2 EARTH LOCATING THE POLAR SATELLITE DATA

The Earth viewing instruments on the NOAA satellites (with one exception) are cross-track scanners; i.e., they scan perpendicular to the direction of movement of the satellite. For each instrument several quantities are important for Earth viewing:

1. The angular field of view for a single observation - instantaneous field of view (IFOV) - not needed to compute Earth locations.

2. The step angle between consecutive observations.

3. The step time.

4. The number of steps.

These values are all measured at the satellite position. A scan angle of zero corresponds to the midpoint of the scan. It may not correspond to an actual IFOV. Values for the scan data for each instrument can be found in Appendix J.

The actual area on the Earth observed in an IFOV or encompassed within the width of a scan swath depends on the height of the satellite above the Earth surface. This height is not constant for two reasons. Even if the satellite were in a perfectly circular orbit, the Earth is approximately an ellipsoid, the radius of which varies by 21.4 kilometers from equator to pole. Additionally, the satellite orbit, while very close to circular, is not exactly so. Since the Earth center is at one

focus of the elliptic orbit, it is displaced to one side of the ellipse. While the combined effects of these two factors must be considered for correct Earth location, for generalized approximations they may be neglected, and the orbit considered to be perfectly circular about a spherical Earth.

Within a single scan, the area on the Earth within an IFOV will increase with the angle from nadir, due to the combined effects of an oblique viewing angle and the curvature of the Earth. The nadir direction is defined by a line through the satellite which is perpendicular to the surface of the Earth ellipsoid. This point of intersection of this line with the ellipsoid is defined as the sub-satellite point (SSP).

Using the satellite position, the scan angle, and any pertinent attitude angles (in Section 2.3), the direction in inertial space in which a given instrument is looking - the Line of Sight (LOS) - is found. The intersection of the LOS with the ellipsoid may then be found. In order to determine the longitude of the intersection, the Greenwich Hour Angle (GHA) must be found for the time of the observation. This is the longitude difference between the Greenwich meridian and the zero longitude of the inertial coordinate system - the Vernal Equinox. Once this is done the exact Earth location which the instrument is viewing (geodetic latitude and longitude) may be found. See Appendix I for the Earth location algorithm**.**

2.3 NAVIGATION AND EARTH LOCATION PROCESSING WITHIN NOAA

To provide accurate satellite navigation data, the Engineering Branch of the Mission Operations Division (MOD), which is part of the Office of Satellite and Product Operations (OSPO), receives a daily set of Inertial Osculating Cartesian orbit parameters for each polar satellite from the Air Force or Navy. This orbit vector is used to generate a predicted User Ephemeris File (UEF) of orbit vectors spaced one minute apart that cover a 10 day time span. This file is created using a COWELL numerical integrator (using Störmer-Cowell formulas) which maintains the one kilometer accuracy of the initial orbit vector (Maury and Brodsky, 1969 and Moore and Beandet, 1973). The UEF is the foundation for all the navigation data produced in MOD. It is utilized to create the TBUS bulletins, the equator crossing information files, the Search and Rescue (SAR) orbit ephemeris files and Level 1b instrument data files. The SAR ephemeris data is provided for use by the U.S. Mission Control Center for Search and Rescue. The level 1B process uses the orbital information in the UEF files to provide Earth located data for the NOAA polar satellite instruments.

The Earth location data provided in the Level 1b process is produced by the Advanced Earth Location Data System (AELDS). This system was initially implemented on September 8, 1992 in the AVHRR Level 1b process and on September 7, 1994 in the TOVS Level 1b process. It is an on-line Earth location process utilizing the scan line time codes to produce Earth location. With the introduction of AELDS the accuracy of the Earth location data in the Level 1b file was improved by 50%.

The Earth location algorithm used to produce the latitude and longitude parameters within the AELDS process are available in Appendix I. The AELDS process provides more than just latitude and longitude information. Given the satellite position and velocity vector and the Greenwich Hour Angle (GHA), AELDS will provide the satellite height and

northbound/southbound flags. Also, given the scan time, stepping time, stepping angle, and number of positions desired; AELDS can also provide the following for each scan point of a specific instrument:

- Solar zenith angle
- Satellite zenith angle
- Solar azimuth angle
- Satellite azimuth angle
- Relative azimuth angle

In order to insure that the Earth location and navigation information provided by MOD lies within acceptable accuracy limits, quality control (QC) operations are performed during and after generation of the data. At present, three types of checks are used:

> 1) Navigation: When the UEF containing predicted satellite position and velocity data is generated; the radius vector is compared to that generated using the elements for the previous seven days of data (delta-R). Generally, these differences remain less than one kilometer for at least 7 to 10 days.
>
> 2) On-line Earth location: An Earth location tolerance check of the satellite subpoint (nadir) location has been integrated into the AELDS process. The subpoint position is calculated by an independent method and compared with the position generated by AELDS. The acceptable value of the difference can be reset and the actual option can be turned on or off. This tolerance check gives the reassurance that the Earth location algorithm is behaving correctly.
>
> 3) Post processing Earth location: An image QC system is used to verify the accuracy of the Earth location data generated using the UEF and appended to the AVHRR instrument raw data in the level 1B files.

Utilization of the above image QC techniques has provided greater insight into the magnitude of Earth location errors as well as the source of some of the errors. To increase the data accuracy, MOD has enhanced the on-line Earth location process (AELDS) to include fixed attitude corrections and TIP clock corrections.

Fixed attitude corrections include corrections for errors such as instrument mounting errors and constant observed errors. An algorithm has been integrated into the Earth location process that accounts for these errors.
The Satellite Operations Control Center (SOCC) maintains the on-board clocks for the NOAA POES satellites. They monitor the accuracy of the clocks and make adjustments whenever needed. The SOCC clock adjustments are made for two reasons:

> 1) The clock has drifted outside the tolerance level and is reset or corrected to over compensate for the error. The clock error is then allowed to drift back through zero until it again exceeds the tolerance.

2) When a leap second is needed at the end of December or June, the TIP clock is adjusted depending on the resultant error when combined with the existing clock error.

MOD maintains clock drift files containing all SOCC corrections. These files are utilized to correct the Earth location data using the instrument scan times and are available to the user community in the following URL: http://www.ospo.noaa.gov/Products/ppp/navpage.html.

Generally, after the fixed attitude corrections and TIP clock corrections, the Earth location error seen in the image data around the satellite subpoint remains within 1 kilometer (specifications for AVHRR are 4-5 kilometers). The error near the limb is expected to be larger, and is often less than 3 kilometers. At any time during the process, these attitude corrections may be turned on or off.

2.3.1 UPDATES TO THE TBUS BULLETIN

The TBUS bulletin is a major source of orbital data for direct readout users. Although, the UEF is used in creating all four parts of the TBUS bulletin, MOD's focus is on Part IV of the bulletin. More information on the TBUS bulletin can be obtained from section 5.1 and Appendix A. MOD, in conjunction with SOCC, has outlined steps to make Part IV more user-friendly and to increase the accuracy of the data (the Brouwer Mean elements). Action has been taken in the following areas:

- The clock correction information found in the comments section of the TBUS bulletin, was provided in a format similar to that in Part IV. The objective is to maintain consistency in the location of the clock correction information so that users can automate their process.

- The equator crossing longitude was added in the spare fields at the end of Part IV.

- Methods for improving the accuracy of the Brouwer Mean elements are under investigation.

- The BROLYD orbit prediction software package currently available to the user community will be implemented on a personal computer. Test runs of the software will be analyzed to determine modifications to the procedure, software, or algorithm that will improve the performance or accuracy. Other propagation packages that could be made available to the users will be examined to determine their usefulness.

2.3.2 NAVIGATION DATA ON THE INTERNET

The MOD Polar Navigation information can be obtained from http://www.ospo.noaa.gov/Products/ppp/navpage.html

- TBUS messages - Both current and limited historical TBUS bulletins are available for each of the operational NOAA polar satellites.

- Brouwer/Lyddane Software Package - This package exists on the network as part of the NOAA Polar Orbiter Data Users Guide and also as Appendix B in this document.

The following entries are a part of the navigation page:

- Graphical orbit locator - This locator can be used to plot the orbit subtrack for NOAA polar orbiters either on a map of the Northern Hemisphere, Southern Hemisphere, or Equatorial Equal Spaced projection. Directions for the use of the locator are included in the page. An example of the plots available through this software are shown in Figures 2.3.2-1 and 2.3.2-2.

- Equator crossing data - Files for each of the operational polar satellites are presented on this page. Current data and limited historical data are available. Each file contains entries for one hundred orbits showing the equator crossing time in the form of a modified Julian day number (Julian day minus 2400000.5) and Gregorian calendar date with time in hours/minutes/seconds. The equator crossing longitude (East) and orbit number are also provided for each entry. An example of the equator crossing data for NOAA-14 is shown in Figure 2.3.2-3 (note that the column and row separators in this figure do not appear in the data itself).

- Earth location algorithm - The Earth location algorithm used in the AELDS software is provided in Appendix I and as a link in the overview from the navigation page.

- TIP Clock Error Database - The file used by the AELDS process to correct Level 1b data for TIP clock errors is available on the navigation page. There are files for each of the operational polar satellites. The files contain entries in chronologically ascending order (newest update first). An example of the contents of the file is given in Figure 2.3.2-4.

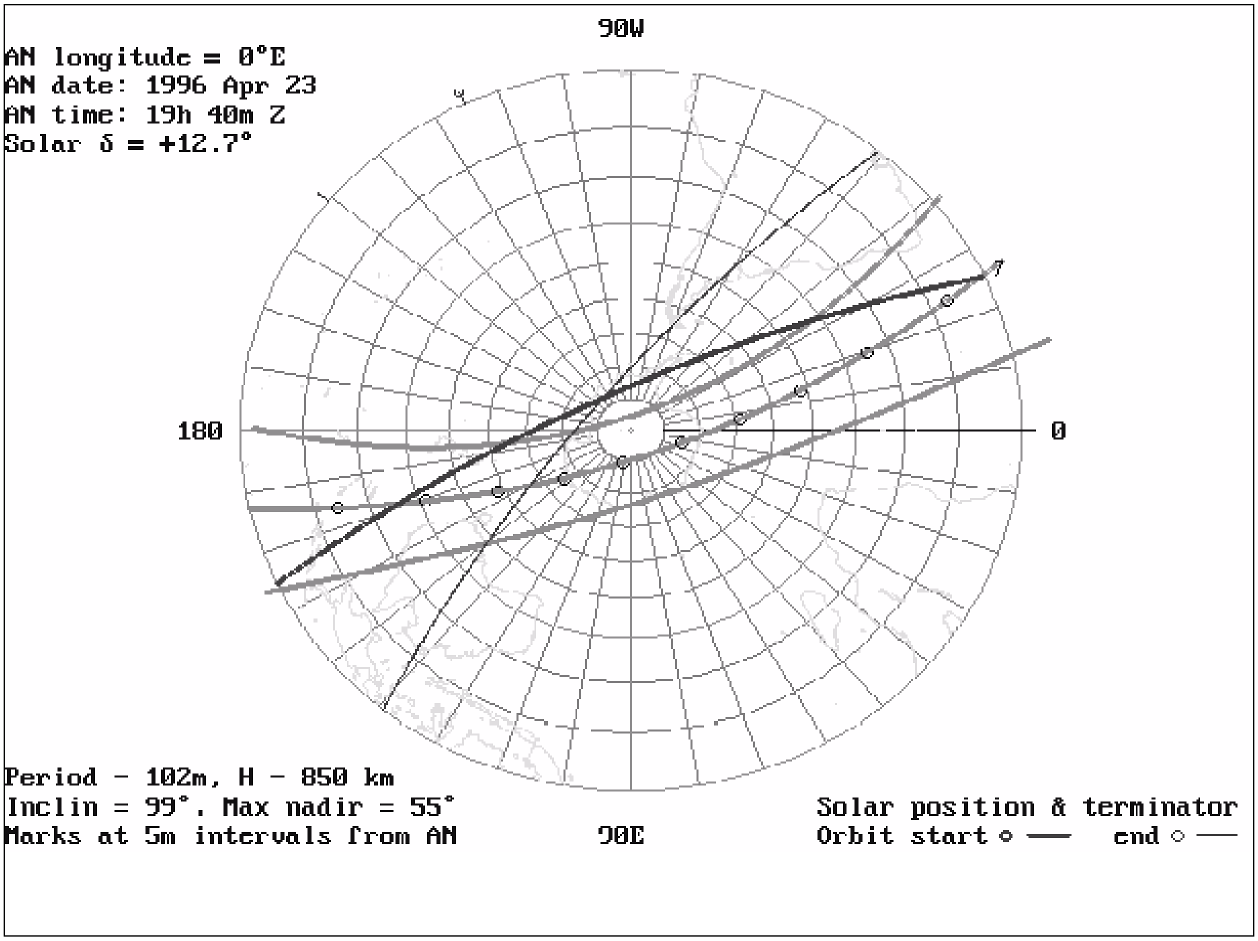

Figure 2.3.2-1. Sample plot available from Graphical orbit locator.

The MOD updates or enhancements to the navigation and Earth location process are on going. For example, an Earth location error database is planned for Level 1b data users. This database will contain Earth location error information based on actual error measurements taken from AVHRR imagery. Every attempt was made to keep the current systems compatible with the prior satellite series.

Figure 2.3.2-2. Sample plot available from Graphical Orbit Locator.

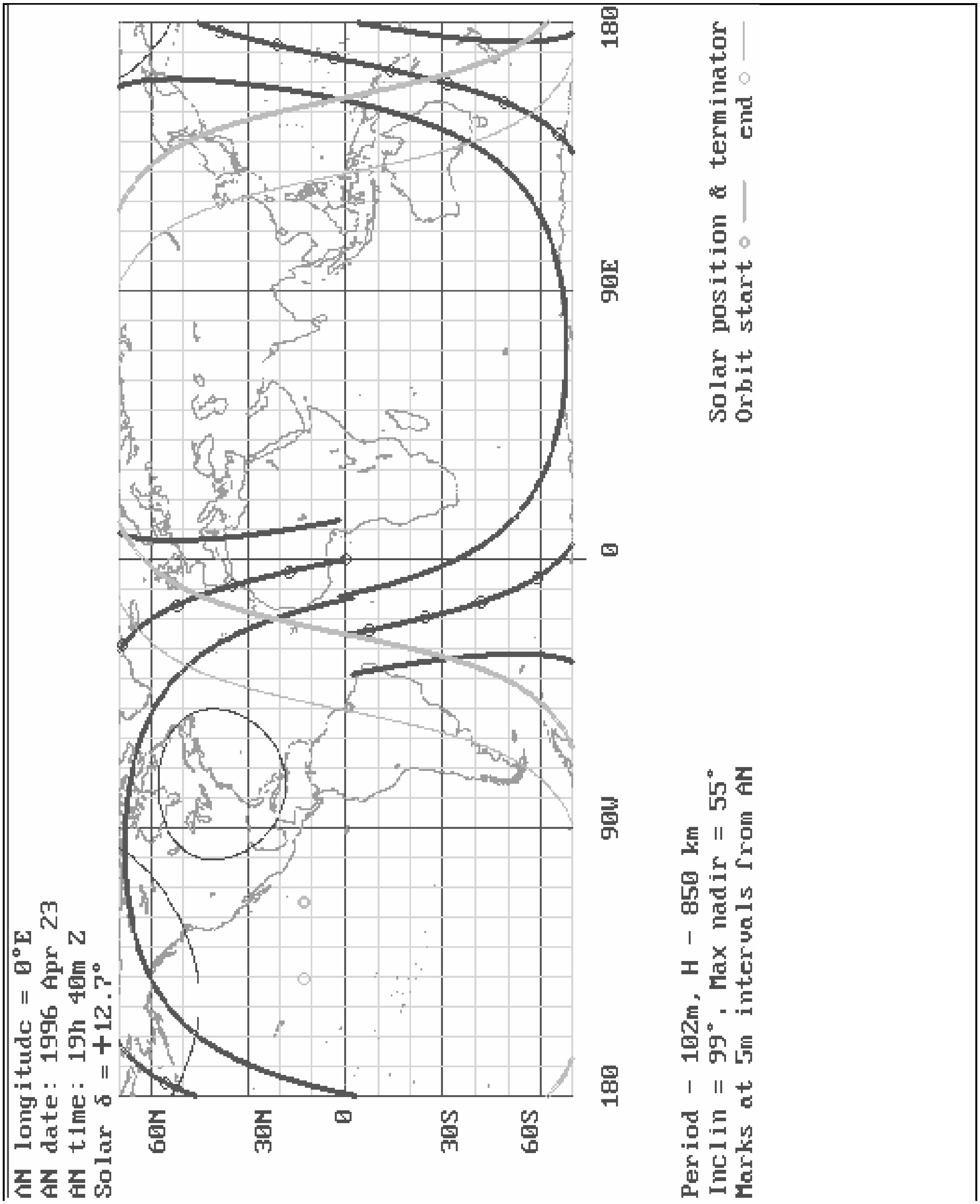

Table 2.3.2-1. Sample NOAA-14 Equator Crossing Data.									
N	**REV NUM**	**EQ.XING.TIME (MJD)**	**Year**	**Month**	**Day**	**Hour**	**Min**	**Second**	**EQ.XING LONGITUDE**
1	6893	0.502049988306D+05	1996	5	1	23	58	18.961	208.8628
2	6894	0.502050697122D+05	1996	5	2	1	40	23.132	183.3453
3	6895	0.502051405938D+05	1996	5	2	3	22	27.303	157.8277
4	6896	0.502052114754D+05	1996	5	2	5	4	31.474	132.3106
5	6897	0.502052823570D+05	1996	5	2	6	46	35.646	106.7937
6	6898	0.502053532386D+05	1996	5	2	8	28	39.817	81.2773
7	6899	0.502054241202D+05	1996	5	2	10	10	43.988	55.7614
8	6900	0.502054950018D+05	1996	5	2	11	52	48.159	30.2455
9	6901	0.502055658835D+05	1996	5	2	13	34	52.331	4.7292
10	6902	0.502056367651D+05	1996	5	2	15	16	56.502	339.2124
11	6903	0.502057076467D+05	1996	5	2	16	59	0.673	313.6947
12	6904	0.502057785283D+05	1996	5	2	18	41	4.844	288.1778
13	6905	0.502058494099D+05	1996	5	2	20	23	9.015	262.6632
14	6906	0.502059202915D+05	1996	5	2	22	5	13.187	237.1478
15	6907	0.502059911731D+05	1996	5	2	23	47	17.358	211.6310
16	6908	0.502060620547D+05	1996	5	3	1	29	21.529	186.1136
17	6909	0.502061329363D+05	1996	5	3	3	11	25.700	160.5959
18	6910	0.502062038180D+05	1996	5	3	4	53	29.872	135.0787
19	6911	0.502062746996D+05	1996	5	3	6	35	34.043	109.5618
20	6912	0.502063455812D+05	1996	5	3	8	17	38.214	84.0453
21	6913	0.502064164628D+05	1996	5	3	9	59	42.385	58.5293

Table 2.3.2-2. Sample NOAA-14 TIP Clock Error Database.					
Spacecraft ID# (NOAA-14)	**Effective Time**		**Clock Error Millisec I5**	**Drift Rate Millisec/Day F7.3**	**Comments (Optional)**
	Year/MM/DD	HH:MM:SS.SSS			
23455	1996/03/19	23:59:00.000	-035	+7.000	
23455	1995/12/31	23:59:59.000	-200	+5.000	-1.0 Leap second Adjustment
23455	1995/08/01	23:59:00.000	-560	+5.000	
23455	1995/01/01	23:59:00.000	-560	+5.000	
23455	1994/01/01	00:01:00.000	000	0.000	

2.4 INTERPOLATING THE LEVEL 1b EARTH LOCATION DATA

The geographic location of AVHRR LAC/HRPT viewed areas presents a problem, since one scan line contains 2048 viewed spots, but only 51 of these are located in the Level 1b data; point 25 being the first located point, and then every fortieth point. The scan is from right to left, when facing in the direction of satellite motion. GAC data are less of a problem, since every eighth

point is located. **It should be noted, however, that the Earth locations given for GAC areas are not in the center of the area**; see Section 2.4.4. All other instruments currently on NOAA satellites have all observed areas located. Since investigators often require the location of all observed points, the locations for points intermediate between those given in the Level 1b data must be obtained by interpolation. The common method is to interpolate linearly separately in latitude and longitude between given points. This study was undertaken to determine the accuracy of such linear interpolation, and if this method should prove insufficiently accurate, to determine an interpolation method which would yield acceptable accuracy.

2.4.1 METHOD

In order to ascertain the accuracy of an interpolation method, it is necessary to know the true location of every point. In order to simplify the computations, a spherical earth was assumed, permitting the use of spherical trigonometry. The radius was taken to be 6371 km, the approximate mean radius of the actual Earth.

The satellite altitude was assumed to be 850 km with an inclination, β, of 99 degrees, and assumed to be in the first quarter orbit (northbound, in the Northern Hemisphere), and only the right (first) half of the scan was considered. These assumptions will have no effect on the results, since the configuration is completely symmetrical. The trace of the scan line on the earth may be assumed to be a great circle, especially since only the portion of a scan which includes at most five Level 1b located points will be examined at one time. The time for this to occur is about 5 milliseconds, during which the sub-satellite point will have moved about 35 meters. The satellite position was therefore taken as fixed, and the 1024 true scan locations calculated, using a scan step of 0.0541 degrees. It should be noted that this scan step, combined with the field of view of the sensor, results in an over sampling of 1.362 samples per field of view; in other words, the LAC data overlap (see Figures 2.4.4-1 and 2.4.4-2). No attempt was made to do any calculations which involved using points on both sides of the nadir point. This is of little consequence, since interpolated positions are most accurate near nadir.

Figure 2.4.1-1. Relationship of sub-satellite point to Earth.

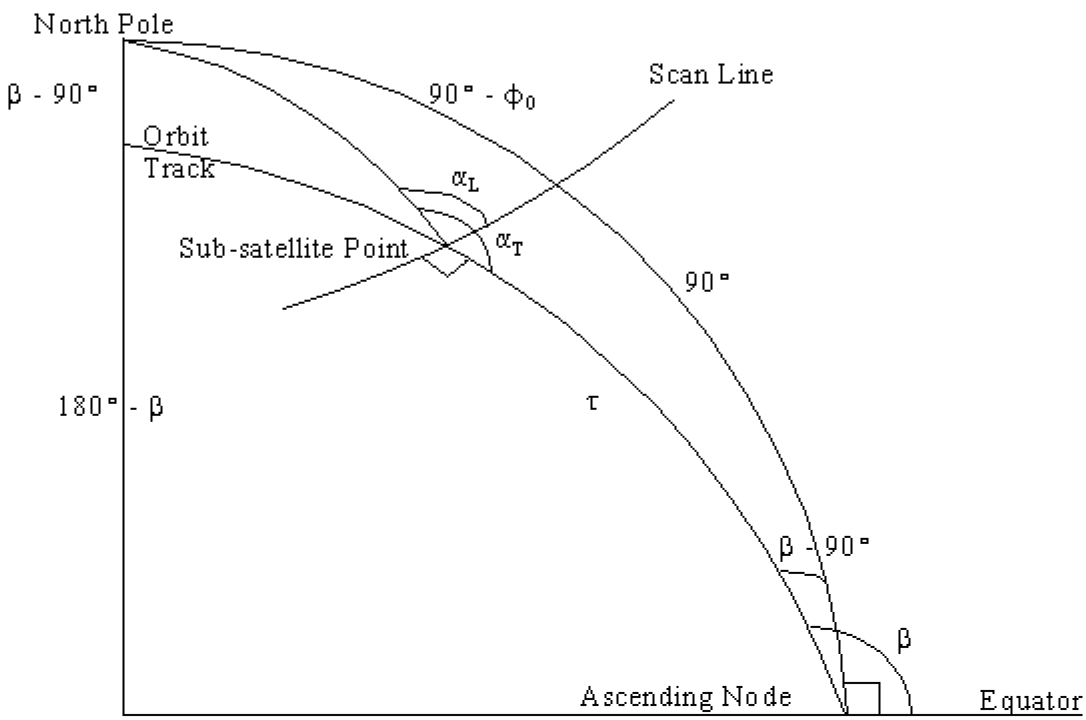

Figure 2.4.1-1. Relationship of sub-satellite point to Earth.

Since all calculations assume instantaneous conditions, it is not necessary to consider Earth rotation. φ_0 is the satellite latitude and zero longitude was taken to coincide with the sub-satellite point. From Figure 2.4.1-1, the angle at the sub-point from north to the sub-track is:

$$\alpha_T = \sin^{-1}\left(\frac{-\cos\beta}{\cos\phi_0}\right)$$

and the angle to the scan line is:

$$\alpha_L = \alpha_T - 90°$$

For a viewing nadir angle (scan angle) σ, the corresponding geocentric arc distance is given by:

$$\theta = \sin^{-1}\left(\frac{R+H}{R}\sin\sigma\right) - \sigma$$

where R is the Earth radius and H is the satellite height (see Figure 2.4.1-2). From the spherical

$$\sin\phi = \sin\phi_0 \cos\theta + \cos\phi_0 \sin\theta \cos\alpha_L$$

triangle shown in Figure 2.4.1-3, the latitude of the viewed point, φ, may be found to be:

and the longitude relative to the sub point by:

$$\cos \Delta\lambda = \frac{\cos \phi_0 \cos \theta - \sin \phi_0 \sin \theta \cos \alpha_L}{\cos \phi}$$

Performing these computations for each scan step, starting at an angle from nadir of half a scan step, defines the true Earth locations to which the interpolated locations will be compared.

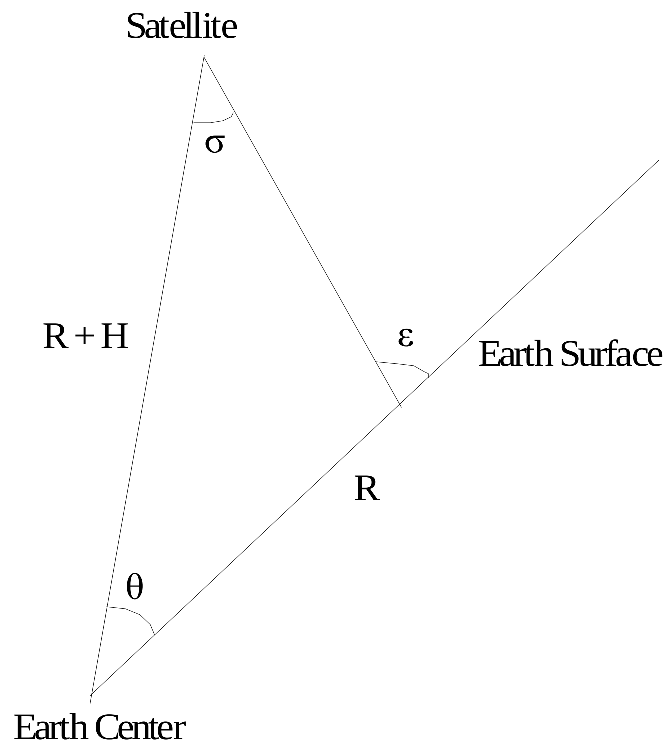

Figure 2.4.1-2. Angular relationship between satellite, surface and Earth Center

2.4.2 RESULTS

The results for linear interpolation are shown in Table 2.4.2-1. The point numbers are those of located points, and the range of scan angles are shown for each group. In each group of 39 interpolated points, the mean error and the maximum error are shown, converted to linear distance in kilometers on the surface of the Earth. For all interpolation methods, the variation with sub-point latitude is small, as would be expected as a result of the spherical assumption. For an ellipsoidal Earth, the variation may be somewhat larger. Near nadir the errors are small, but become completely unacceptable by the time the most limbward pair of points are reached.

Table 2.4.2-1. Errors for Linear Interpolation Between Adjacent Located AVHRR Points for Latitude = 40^0.				
Point Number	**Scan Angle Range (degrees)**		**Mean Distance (km)**	**Maximum Distance (km)**
	From	**To**		
1- 2	-54.073	-51.909	2.5082	3.8583
2 - 3	-51.909	-49.745	1.7198	2.6449
3 - 4	-49.745	-47.581	1.2518	1.9248
4 - 5	-47.581	-45.417	0.9497	1.4604
5 - 6	-45.417	-43.253	0.7427	1.1422
6 - 7	-43.253	-41.089	0.5944	0.9142
7 - 8	-41.089	-38.925	0.4844	0.7450
8 - 9	-38.925	-36.761	0.4004	0.6159
9 - 10	-36.761	-34.597	0.3348	0.5150
10 - 11	-34.597	-32.433	0.2825	0.4346
11 - 12	-32.433	-30.269	0.2401	0.3694
12 - 13	-30.269	-28.105	0.2052	0.3157
13 - 14	-28.105	-25.941	0.1760	0.2708
14 - 15	-25.941	-23.777	0.1513	0.2327
15 - 16	-23.777	-21.613	0.1301	0.2002
16 - 17	-21.613	-19.449	0.1118	0.1719
17 - 18	-19.449	-17.285	0.0957	0.1471
18 - 19	-17.285	-15.121	0.0814	0.1252
19 - 20	-15.121	-12.957	0.0685	0.1054
20 - 21	-12.957	-10.793	0.0569	0.0876
21 - 22	-10.793	-8.629	0.0463	0.0712
22 - 23	-8.629	-6.465	0.0365	0.0561
23 - 24	-6.465	-4.301	0.0274	0.0422
24 - 25	-4.301	-2.137	0.0193	0.0297

The accuracy is improved by a three point interpolation in latitude and in longitude, using the Lagrangian interpolation algorithm (see Section 2.4.3 for a discussion of this algorithm). The use of the Lagrangian method does not seem to be critical, and any three point interpolation seems to be equally good. As shown in Table 2.4.2-2, the accuracy near nadir becomes even better, while the limbward interval improves markedly, the maximum error being only 2/3 km.

Table 2.4.2-2. Errors for Lagrangian Interpolation Between Three Adjacent Located AVHRR Points at Latitude = 40^0.				
Point Number	**Scan Angle Range (degrees)**		**Mean Distance (km)**	**Maximum Distance (km)**
	From	**To**		
1- 2	-54.073	-51.909	0.4251	0.6758
2 - 3	-51.909	-49.745	0.2495	0.3961
3 - 4	-49.745	-47.581	0.1598	0.2534
4 - 5	-47.581	-45.417	0.1088	0.1724
5 - 6	-45.417	-43.253	0.0776	0.1229
6 - 7	-43.253	-41.089	0.0574	0.0908
7 - 8	-41.089	-38.925	0.0436	0.0691
8 - 9	-38.925	-36.761	0.0340	0.0538
9 - 10	-36.761	-34.597	0.0270	0.0428
10 - 11	-34.597	-32.433	0.0219	0.0346
11 - 12	-32.433	-30.269	0.0180	0.0285
12 - 13	-30.269	-28.105	0.0150	0.0237
13 - 14	-28.105	-25.941	0.0127	0.0201
14 - 15	-25.941	-23.777	0.0109	0.0172
15 - 16	-23.777	-21.613	0.0094	0.0149
16 - 17	-21.613	-19.449	0.0082	0.0130
17 - 18	-19.449	-17.285	0.0073	0.0116
18 - 19	-17.285	-15.121	0.0066	0.0104
19 - 20	-15.121	-12.957	0.0060	0.0094
20 - 21	-12.957	-10.793	0.0055	0.0087
21 - 22	-10.793	-8.629	0.0051	0.0081
22 - 23	-8.629	-6.465	0.0048	0.0076
23 - 24	-6.465	-4.301	0.0046	0.0073

There remain the 24 scan points before the first located point (23 points after the last located point) the locations of which cannot be interpolated, but must be extrapolated. Table 2.4.2-3 presents the values extrapolated from the first three located points. Since the errors change rapidly, values are given for each point. In addition to the total error in kilometers, the individual errors in latitude and longitude are also presented. The error for point 25 is exactly zero, since this is a located point. The location errors grow rapidly, being over 5 km for the limbward point

Table 2.4.2-3. Errors for Lagrangian Interpolation From Three Limbward Located AVHRR Points at Latitude = 40^0.				
LAC Point	**Scan Angle (degrees)**	**Latitude Error (degrees)**	**Longitude Error (degrees)**	**Distance Error (km)**
1	55.425	0.0056	-.0633	5.3122
2	55.371	0.0052	-.0588	4.9389
3	55.317	0.0048	-.0546	4.5818
4	55.263	0.0045	-.0505	4.2403
5	55.209	0.0041	-.0466	3.9140
6	55.155	0.0038	-.0429	3.6026
7	55.101	0.0035	-.0394	3.3055
8	55.047	0.0032	-.0360	3.0225
9	54.993	0.0029	-.0328	2.7531
10	54.939	0.0026	-.0297	2.4969
11	54.884	0.0024	-.0268	2.2535
12	54.830	0.0021	-.0241	2.0226
13	54.776	0.0019	-.0215	1.8038
14	54.722	0.0017	-.0190	1.5968
15	54.668	0.0015	-.0167	1.4012
16	54.614	0.0013	-.0145	1.2166
17	54.560	0.0011	-.0124	1.0428
18	54.506	0.0009	-.0105	0.8794
19	54.452	0.0008	-.0086	0.7260
20	54.398	0.0006	-.0069	0.5824
21	54.343	0.0005	-.0053	0.4483
22	54.289	0.0003	-.0039	0.3232
23	54.235	0.0002	-.0025	0.2070
24	54.181	0.0001	-.0012	0.0994
25	54.127	0.0000	0.0000	0.0000

Table 2.4.2-4 shows the same information except with the extrapolation done with a five point Lagrangian algorithm. This shows a marked improvement, with the maximum error being only slightly greater than 1 km. This would probably be sufficiently accurate for almost all purposes. It should be noted that while the nominal Earth field of view of AVHRR is 1.1 km, this is true only at nadir. The limbwardmost coverage is 6.5 x 2.3 km. A 1 km location error is thus much less than the size of a field of view.

Table 2.4.2-4. Errors for Lagrangian Interpolation From Five Limbward Located AVHRR Points at Latitude = 40^0.				
LAC Point	**Scan Angle (degrees)**	**Latitude Error (degrees)**	**Longitude Error (degrees)**	**Distance Error (km)**
1	55.425	0.0014	-.0121	1.0231
2	55.371	0.0013	-.0111	0.9388
3	55.317	0.0012	-.0102	0.8595
4	55.263	0.0011	-.0093	0.7850
5	55.209	0.0010	-.0085	0.7150
6	55.155	0.0009	-.0077	0.6493
7	55.101	0.0008	-.0070	0.5878
8	55.047	0.0007	-.0063	0.5302
9	54.993	0.0007	-.0056	0.4764
10	54.939	0.0006	-.0050	0.4261
11	54.884	0.0005	-.0045	0.3793
12	54.830	0.0005	-.0040	0.3358
13	54.776	0.0004	-.0035	0.2953
14	54.722	0.0004	-.0031	0.2577
15	54.668	0.0003	-.0026	0.2230
16	54.614	0.0003	-.0023	0.1909
17	54.560	0.0002	-.0019	0.1613
18	54.506	0.0002	-.0016	0.1341
19	54.452	0.0002	-.0013	0.1091
20	54.398	0.0001	-.0010	0.0862
21	54.343	0.0001	-.0008	0.0654
22	54.289	0.0001	-.0006	0.0465
23	54.235	0.0000	-.0003	0.0293
24	54.181	0.0000	-.0002	0.0139
25	54.127	0.0000	0.0000	0.0000

2.4.3 LAGRANGIAN INTERPOLATION

Given a set of n coordinates, x_i, y_i, where i = 1 to n, a general value of y is given by

$$y = \sum_{i=1}^{n} y_i L_i$$

Where

$$L_i = \prod_{j=1,\, j \neq i}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

x being the value corresponding to the y which is being sought (the symbol ϑ indicates a product). It should be noted that the given x points must all be different. However, the points need not be equally spaced, and it is not necessary that they be in order (Meeus, 1991).

A schematic computer program for performing Lagrangian interpolation follows:

```
n                                          Number of input points
x(1),y(1)  x(2),y(2),  ...  x(n),y(n)        Input points
x0                                         X for which Y is to be interpolated

y0=0                                        Interpolated value of Y
for i=1 to n
  L=1
  for j=1 to n
    if j  ≠  i then L=L*(x0-x(j))/(x(i)-x(j))
  next j
  y0=y0+L*y(i)
next i
```

2.4.4 LOCATION OF GAC SPOTS

GAC values are calculated on board the satellite by the following procedure (paraphrased from the Advanced TIROS-N Program, Programming and Control Handbooks for NOAA-KLM and NOAA-N, Section 5.5.3.2.3):

Processed GAC earth data is derived from the earth view portion of every third AVHRR scan line. The starting AVHRR scan is not specified, but the GAC lines are tagged with the times of the AVHRR scans from which they are derived.

The data for each of the five AVHRR channels is processed in accordance with the following five sample-averaging algorithms:

(1) Select only every third AVHRR scan line for data processing. Start with the first AVHRR data sample in the selected scan line.

(2) From the selected scan line, obtain 5 contiguous AVHRR data samples.

(3) Retain the data from the first four samples, and discard the data from the fifth sample.

(4) Form a sum by adding together the data from samples 1, 2, 3 and 4. Form the sum in 12-bit precision.

(5) Divide the sum by four to obtain an "averaged" GAC data word. Round the quotient to 10 bits.

(6) Repeat steps (2) through (5), starting with the next AVHRR data sample, for a total of 409 times to generate the required GAC data words for one scan line. Since each AVHRR line includes 2048 data samples, the final 4 samples of the scan line will be skipped.

(7) Repeat steps (1) through (6) for all the AVHRR scan lines.

Thus, the first AVHRR data sample of each selected scan line is averaged into the firs GAC sample of its line, but the last four AVHRR samples of the line are skipped.

The earth location of the centers of 51 selected AVHRR data samples out of 4028 from each scan line are calculated by the Advanced Earth Location Data System during the preprocessing of the data on the ground, and they are included in the level 1B AVHRR data sets. The 25th data sample of a scan and every 40th thereafter are selected. These same earth locations are included in GAC level1B data sets for certain GAC samples. For an earth located GAC spot, the earth location matches the missing AVHRR sample from the 5 used in generating that GAC spot. The 5th GAC data sample of each GAC line and every 8th sample thereafter are earth located in level 1B.

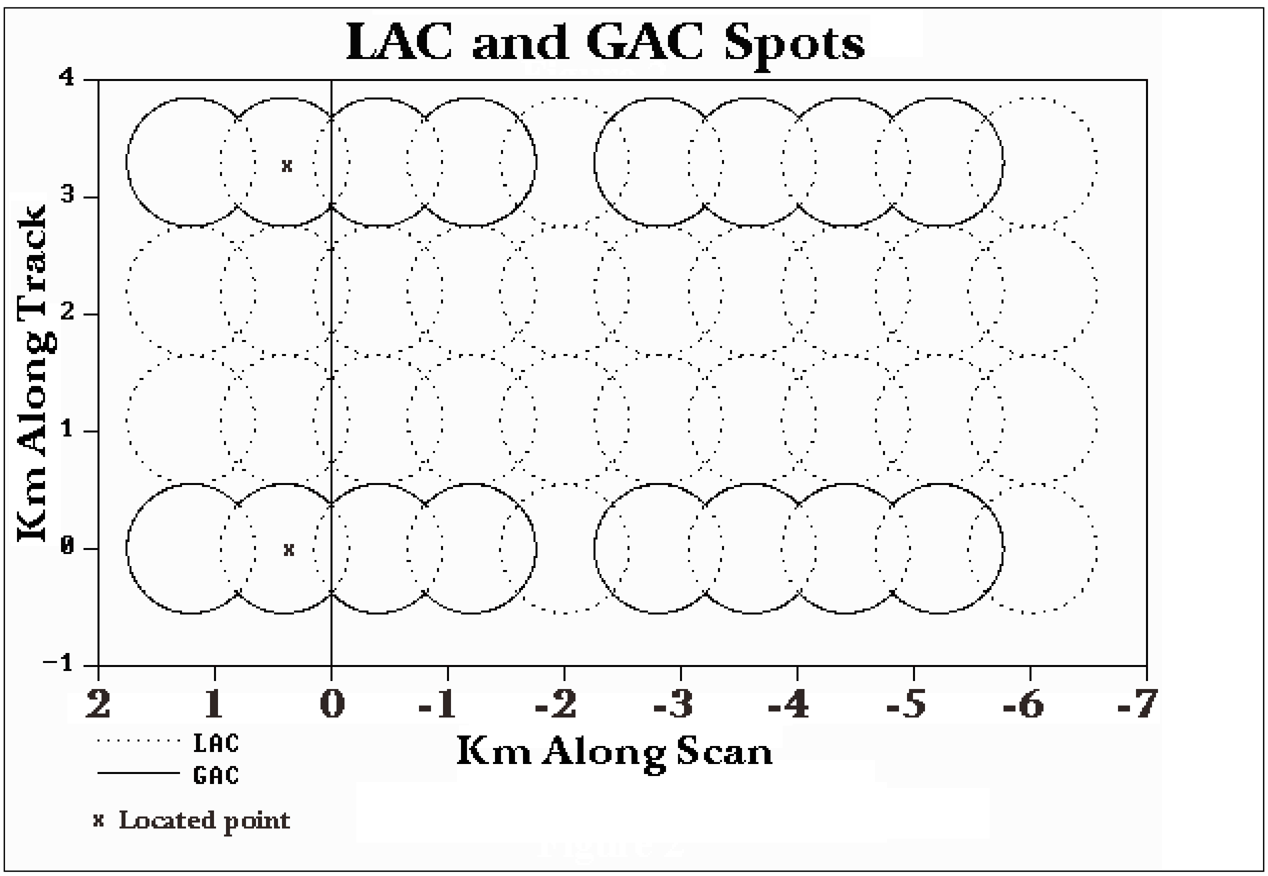

Figure 2.4.4-1. Earth Location of LAC and GAC spots near nadir.

As shown in figure 2.4.4-1, the earth location given for a located GAC spot is NOT that of the center of the spot, the location of the center of the skipped fifth AVHRR spot of the five which constitute a GAC.

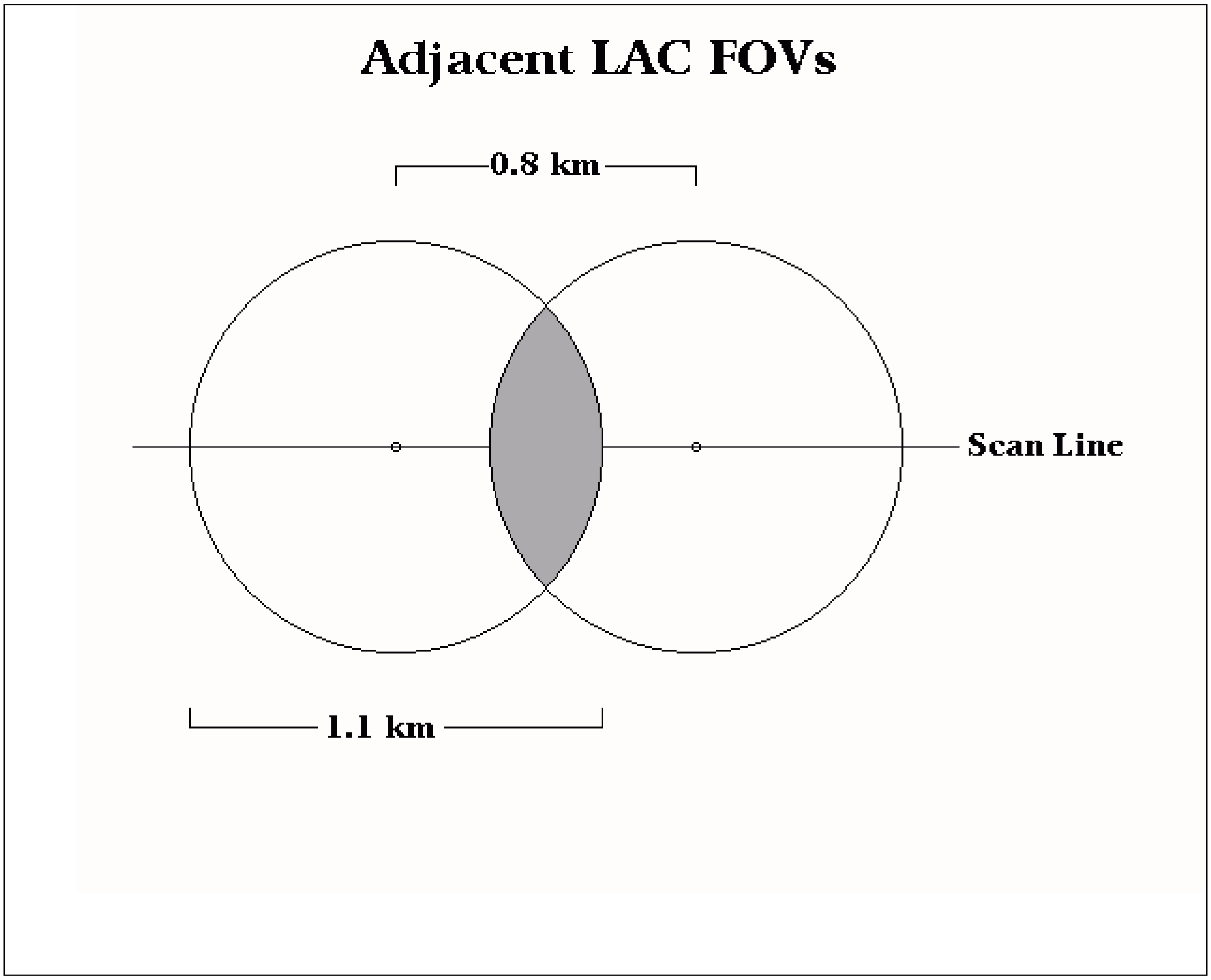

Figure 2.4.4-2. Position of the two LAC spots that straddle nadir.

Figure 2.4.4-2 shows the two AVHRR spots which straddle the nadir. At this position, the Earth location is displaced 2 kilometers from the center of the corresponding GAC spot (see figure 2.4.4-1). At large angles from nadir, the displacement will be larger, but since the Earth field of view will also be stretched in the scan direction (to 6.5 km for the limbmost AVHRR position), the proportional difference will remain the same.

Figure 2.4.4-2 shows the two LAC spots which straddle nadir. At this position, the Earth location given is displaced 0.4 km from the center of the corresponding GAC spot. At large

nadir angles, the displacement will be larger, but since the Earth field of view will also be stretched in the scan direction (to 6.5 km for the limbwardmost LAC position), the proportional difference will remain the same.