Duplicate

NUREG/CR-1389

Estimating Water Equivalent Snow Depth from Related Meteorological Variables

Prepared by L. T. Steyaert, S. K. LeDuc, N. D. Strommen
M. L. Nicodemus, N. B. Guttman

National Oceanic and Atmospheric Administration

Prepared for
U. S. Nuclear Regulatory Commission

Available from

GPO Sales Program
Division of Technical Information and Document Control
U. S. Nuclear Regulatory Commission
Washington, D. C. 20555

and

National Technical Information Service
Springfield, Virginia 22161

NUREG/CR-1389

Estimating Water Equivalent Snow Depth from Related Meteorological Variables

Manuscript Completed: February 1980
Date Published: May 1980

Prepared by
L. T. Steyaert, S. K. LeDuc, N. D. Strommen
M. L. Nicodemus, N. B. Guttman

National Climatic Center
National Oceanic and Atmospheric Administration
Asheville, NC 28801

Prepared for
Division of Siting, Health and Safeguards Standards
Office of Standards Development
U.S. Nuclear Regulatory Commission
Washington, D.C. 20555
NRC FIN No. B1080

Abstract

Engineering design must take into consideration natural loads and stresses caused by meteorological elements, such as, wind, snow, precipitation, temperature, etc. The purpose of this study was to determine a relationship of water equivalent measurements to meteorological variables. Several predictor models were evaluated for use in estimating water equivalent values. These models include linear regression, principal component regression, and non-linear regression models.

The non-linear models seem to have preference over the other models. They are used to obtain water equivalent for a denser network of meteorological stations where predictor variables are available, but which have no water equivalent measurements. The possibility of superior performance of some models developed in foreign areas with consistently greater snow loads than the United States is noted.

Linear, non-linear and Scandanavian models are used to generate annual water equivalent estimates for approximately 1100 cooperative data stations. These estimates are used to develop probability estimates of snow load for each station. Map analyses for 3 probability levels are presented.

CONTENTS

ILLUSTRATIONS

ILLUSTRATIONS (cont'd)

TABLES

I. INTRODUCTION

In the past few years there has been a great deal of interest generated in updating extreme snow load estimates for structural design. The need for revision of design values has been demonstrated by several structural failures in the past couple of winters due in part to excessive snow loads on the roofs. The only studies which provided probability estimates of water equivalent and snow load on the ground for the continental U.S. were done by Thom (14) and ANSI (15). Maps were developed of snow load on the ground for mean recurrence intervals of 2, 10, 25, 50 and 100 years based on observed annual maximum water equivalent measurements from 1952-62 taken at first order Weather Bureau stations. He does not attempt to expand the set of water equivalents by using meteorological data from cooperative observing stations to estimate water equivalents, but he does allude to the potential benefits of such an expansion. These snow load maps have been used as a guide for developing engineering design criteria.

The objective of this study is divided into two phases: Phase I is to expand on Thom's work by examining water equivalent relationships with other meteorological variables by using data collected at the first order stations since the winter season of 1952-53. These relationships will be used to derive water equivalent estimates for numerous cooperative observing stations in the area of consideration. It is conceivable that meaningful relationships which estimate water equivalent from other related meteorological variables can be developed and used to provide estimated water equivalents at first order stations prior to 1952 or for a relatively dense network of cooperative weather reporting stations.

Phase II examines the annual extreme water equivalents and determines which distribution(s) best describe these data. These distributions are used to derive probability estimates of water equivalent and snow load values on the ground for selected mean recurrence intervals.

II. DATA

The data used in this study consist of daily meteorological observations from approximately 83 first order Weather Bureau stations and approximately 1100 cooperative stations in the northeastern quarter of the United States for the winter seasons November-April, 1952-53 through 1978-79. These data include daily measures of water equivalent at the first order stations and depth of snow on the ground, maximum and minimum temperatures, precipitation, and 24-hour snowfall at both first order and cooperative stations. Note that some stations were missing data for some years.

The quality of water equivalent and snow depth measurements for both data sets is influenced by several factors: (1) snow cover may vary considerably within a short distance due to drifting making determination of a representative depth and water equivalent exceedingly difficult, (2) synoptic situations and local topography may significantly affect snowfall characteristics from one area to another, (3) unrepresentative or inconsistent water equivalent measurements can result from exposures at airport sites and from changing station locations, (4) observational and/or recording errors can occur, and

(5) the approximate 10:1 ratio of new snow depth to water equivalent is in some cases applied to new and old snow depth without measuring water equivalent.

III. ANALYSES

A. Phase I - Analysis of First Order Station Data

1. Discussion

The basic variables were grouped by station and winter season for the general period of record 1952-1976 and further stratified into strings. A string is defined as the set of consecutive, daily observations which begin with the occurrence of two inches or more of snow on the ground and end with the next occurrence of less than two inches of snow on the ground. This permits maximum water equivalent and maximum snow depth to be readily associated with previous meteorological conditions. A screening process was developed which eliminated obviously erroneous observations, such as water equivalent measurements which are greater than or equal the snow depth or are not physically supported by associated meteorological measurements.

Two sets of water equivalent data were selected and analyzed at each location. The primary set (W_X) consists of the annual maximum water equivalent and the associated string of observations. The secondary set of data (S_X) consists of annual maximum snow depth, associated water equivalent and observations within the string associated with the annual maximum snow depth. Even though water equivalent and snow depth may not maximize on the same date, the strings associated with the two parameters are generally in common. Some instances were noted, however, when annual maximum water equivalent and annual maximum snow depth did not maximize in the same string (Figure 1).

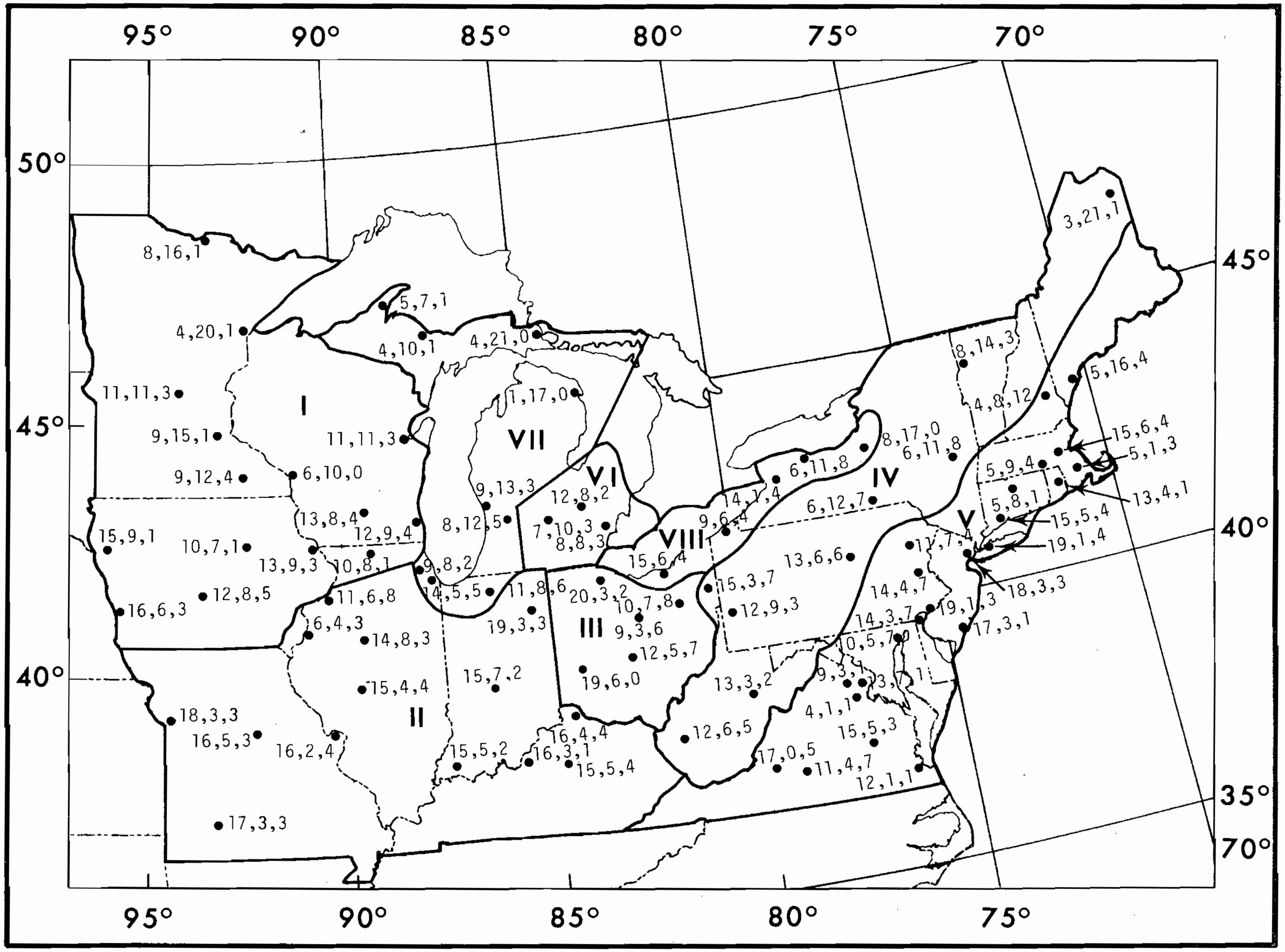

Figure 1. The number of times that annual maximum depth and annual maximum water equivalent occurred on the same date, within the same string, but not on the same date, or within different strings for the sample from the period 1952-1976.

Water equivalent, density and depth, as well as derived variables were analyzed for linear relation to each other. Variables demonstrating significant, physical relationships to water equivalent were considered as potential predictors for ordinary least squares analysis, regression on principal components and non-linear regression analysis.

The analysis on seasonal data for the period 1952-76 was performed in two stages. Stage 1 examined relationships between new snow and associated weather data such as temperature, rainfall, etc., with annual maximum water equivalent data from one location per state. In stage 2, variables in the string of daily observations associated with each annual maximum water equivalent and snow depth during the period 1952-76 were analyzed for about 83 stations. The variables included cumulative precipitation, snow depth, number of snowfalls and the number of days with snow on the ground. This procedure permitted a detailed analysis for a sample to sort out problems and allowed more confidence to be placed in the final results. It also determined areas of homogeneous response, thus enabling a regional aggregation of the data.

Because of the multicollinear relationships among predictor variables, principal component analysis was considered and evaluated at individual locations and also for regions. However, these models had poor predictive capabilities and are not discussed any further.

Two regression models proved to provide the best results for predicting water equivalents. The first is the ordinary least squares regression model of the following general form:

$$u = \beta_o + \sum_i \beta_i x_i + \varepsilon \qquad (1)$$

where:

u is the natural logarithm of water equivalent,

β_o is the regression constant,

β_i is the ith regression coefficient associated with the ith predictor variable, X_i, and

ε is the error term.

Σ is an additive operation.

The relations of cumulative precipitation, snow depth, snowfall and number of snowfalls within a string associated with the maximum water equivalent all exhibit a non-linear relation (Figures 2, 3, 4). Therefore, a non-linear regression model was considered as a possible method for developing the best predictive relationship. The non-linear regression equation permits minimizing the errors without linearizing the relationship between density, depth and water equivalent.

The second model is a non-linear regression equation of the general form:

$$w = \alpha_o \prod_i \alpha_i X_i + \varepsilon \qquad (2)$$

where:

w is the untransformed water equivalent,

α_o, α_1, $\alpha_2 \ldots, \alpha_n$ are regression parameters,

X_1, $X_2, \ldots$, X_n are predictor variables, and

ε is the error term.

π indicates a multiplicative operation.

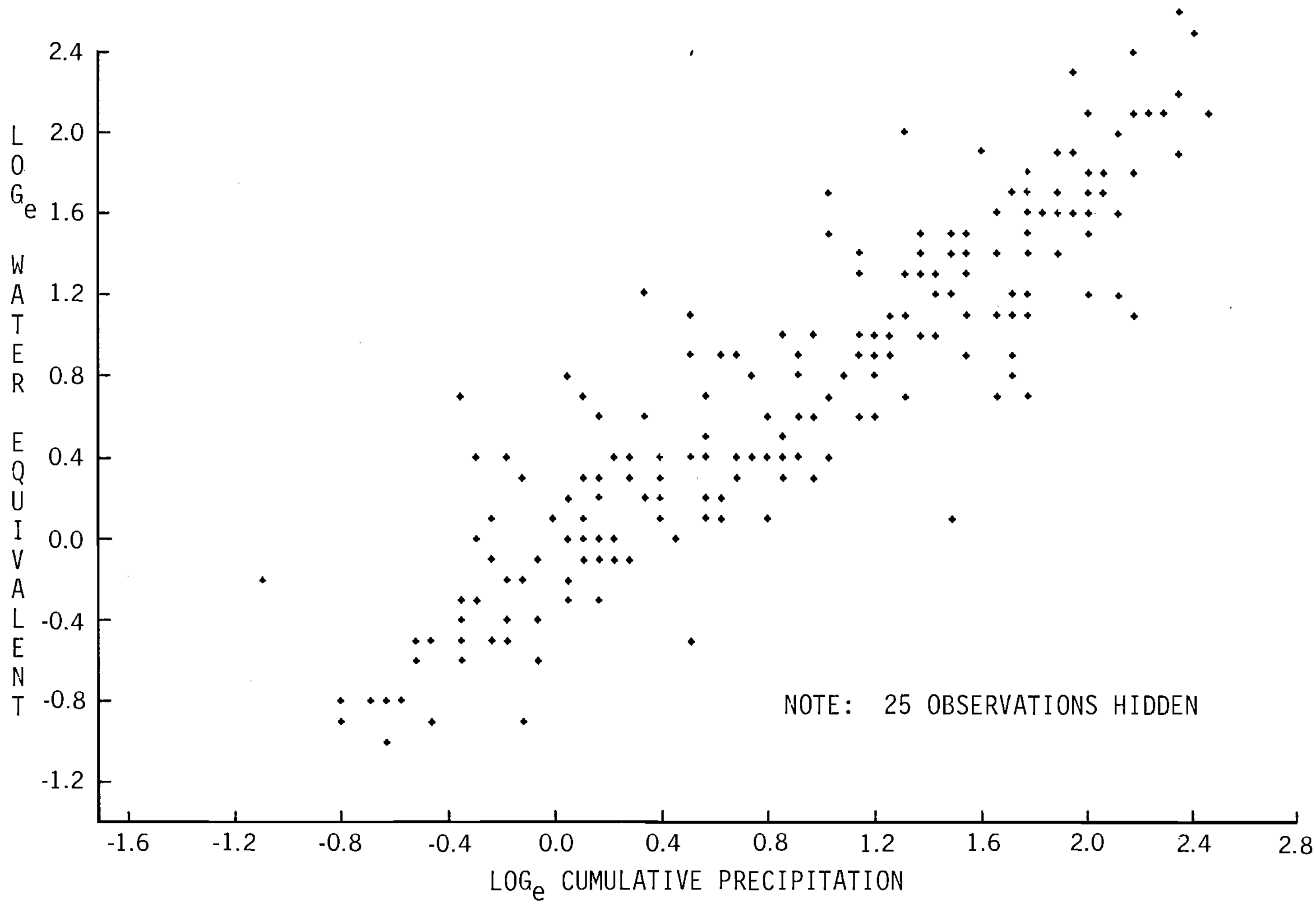

Figure 2. The natural logarithm of water equivalent vs the natural logarithm of cumulative precipitation for Region 7.

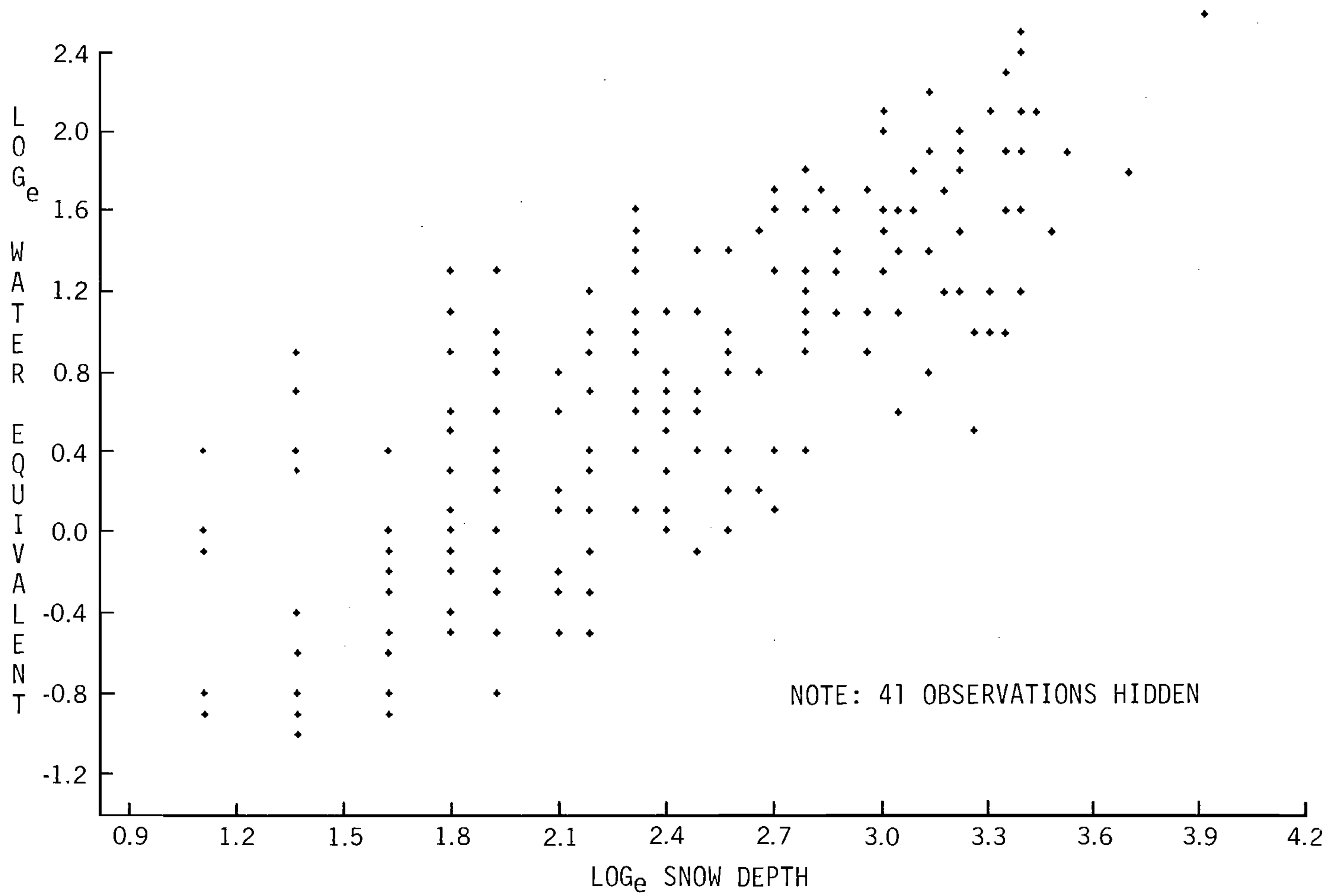

Figure 3. The natural logarithm of water equivalent vs natural logarithm of snow depth for Region 7.

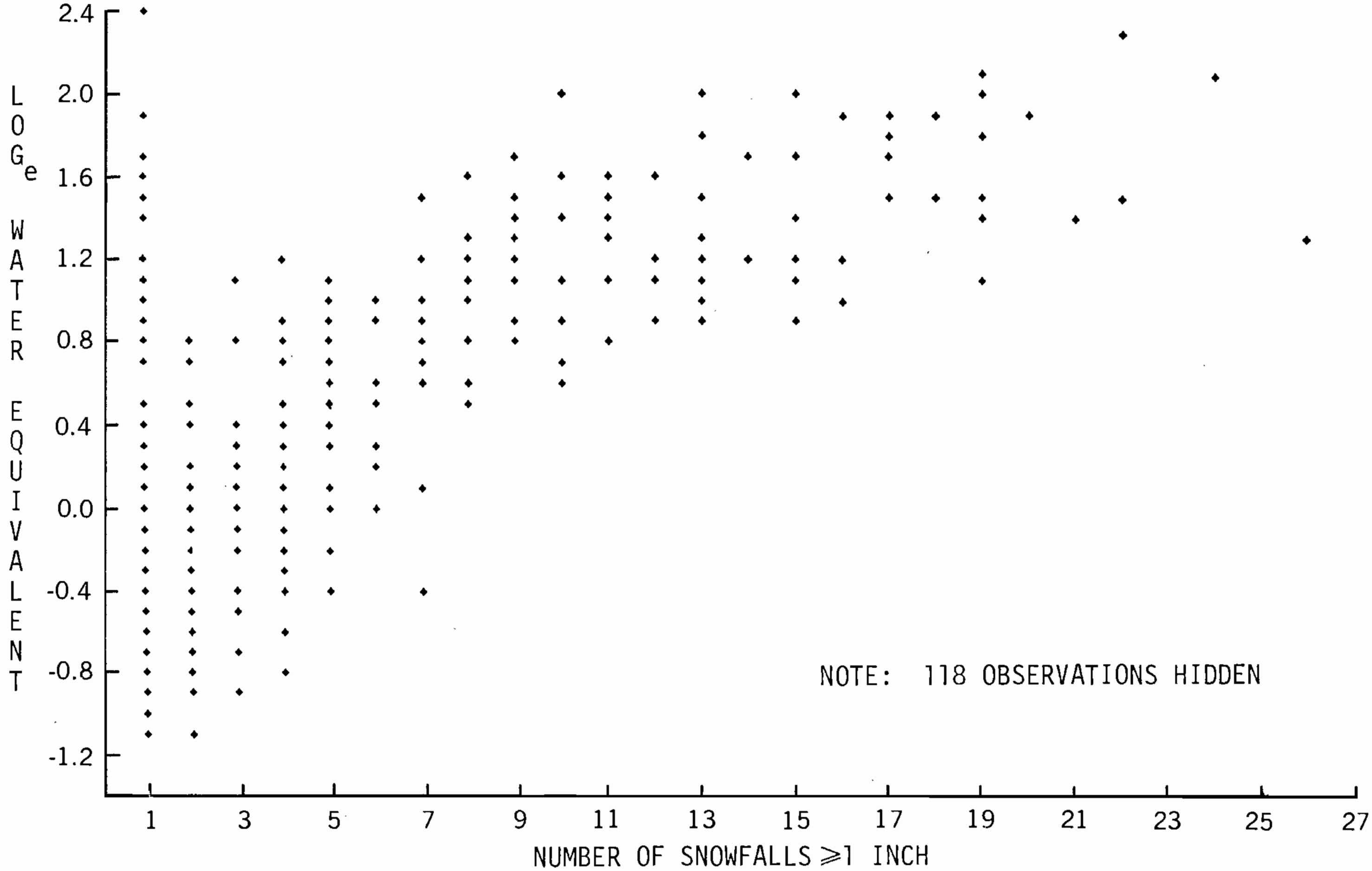

Figure 4. Natural logarithm of water equivalent vs the number of snowfalls greater than or equal to 1 inch.

2. Results

a. The "Pilot" Study

This analysis determined the degree to which characteristics of individual snowfall events could be resolved by daily data. The relationships among temperatures, precipitation, water equivalent, and snowpack density were investigated. Some limitations in the data were confirmed. For example, in many cases snowfall and precipitation cannot be clearly associated with specific snow depth and water equivalent due to differences in observation times of the two variables introducing "noise" into the relationship between the previous day's precipitation and the water equivalent. Similar complications result between the day-to-day change in water equivalent (or snow depth) and the daily snowfall.

Daily minimum and maximum temperatures are not, in general, linearly related to water equivalent. Snowfalls resulting in large water equivalent amounts are generally associated with temperatures which tend to cluster about 30°F, but this is not without exceptions. Neither squared departure from average of maximum temperature nor minimum temperature is related to the water equivalent amount. Subsequent analysis of mean temperature over several days and of various temperature indices demonstrated no consistent, usable temperature relationships to water equivalent. This conclusion is in agreement with United States Weather Bureau (18).

It was concluded that incompatibility of observation times and uncertainty of the times of occurrence for minimum and maximum temperatures are, in part, responsible for the above problems. These and other sources of error within water equivalent observations led to the decision not to consider any type of daily budgeting process to account for changes in water equivalent, evaporation, and maturing of the snowpack.

b. Alternative methods.

The basic underlying assumption of this study is that regression analysis relating meteorological data to water equivalent is the most desirable method for estimating extreme water equivalents from cooperative station data. The water equivalent of a snowpack is not only a function of snow depth but also a function of many highly related factors, such as the number and density of individual snowfalls, drifting, settling of the snow, evaporation and sublimation from the snowpack, and rainfall on mature snow. The above assumption will be evaluated by comparison of extreme water equivalent estimates to independent data and by using methods derived by others (6).

In addition to the form of the predictive model, there is the related decision to determine the most desirable (and feasible) method of applying any relationship to cooperative data which do not have any water equivalent data for verification. Predictive relationships can be developed either with the set of water equivalents observed concurrently with the annual maximum snow depths (S_X) or with the data associated with set of annual maximum water equivalents themselves (W_X). This is an important consideration because annual maximum water equivalent and annual maximum snow depth do not always maximize on the same date particularly for stations which experience extended periods of snow on the ground. Figure 5 shows the average number of days between annual maximum water equivalent and annual maximum snow depth. Some examples of large water equivalent differences between the W_X and S_X data sets are presented in Table 1. Note that the water equivalent associated with the

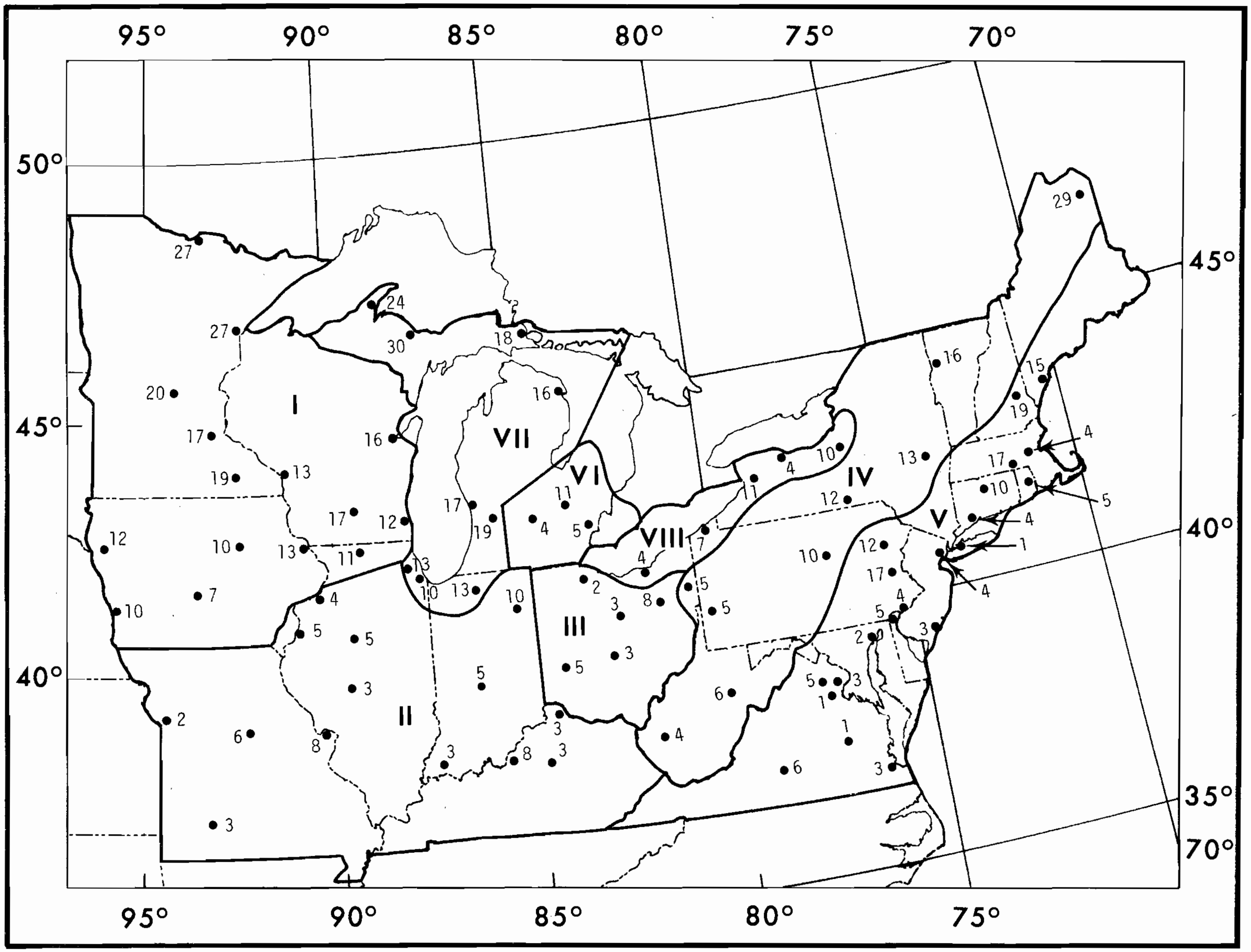

Figure 5. The average number of days between maximum water equivalent and maximum snow depth for those cases in the same string.

TABLE 1

Comparison of Water Equivalents in the W_x and S_x Sets

Name	Absolute Max. W_x	$W_x(S_x)$			
Binghamton, NY	5.9	5.9(3.8)	5.9(2.2)	4.4(2.6)	4.2(2.5)
Elkins, WV	4.2	4.2(2.8)	2.2(1.5)		
Washington, DC	2.7	2.7(2.0)	2.4(0.9)	2.0(1.9)	
Caribou, ME	11.0	11.0(4.5)	9.9(6.0)	9.1(7.7)	8.7(5.3)
Buffalo, NY	7.8	7.8(7.6)	6.2(2.8)	4.0(2.2)	3.7(1.2)
Albany, NY	4.9	4.9(4.8)	4.5(4.8)	3.4(3.2)	2.4(2.2)
Boston, MA	4.8	3.2(2.2)	2.5(2.1)	2.5(1.5)	2.5(1.0)
Hartford, CT	3.6	3.4(2.9)	3.0(2.7)	2.8(2.8)	2.3(1.5)
Burlington, VT	8.9	8.9(2.1)	5.8(3.4)	5.7(4.6)	2.7(1.5)
Concord, NH	6.9	6.9(1.8)	6.5(3.0)	5.4(3.6)	5.0(4.8)
Portland, ME	9.8	9.8(4.1)	7.8(6.6)	6.1(4.8)	5.2(3.2)
Providence, RI	3.8	3.3(1.8)			
Rochester, NY	6.4	6.4(6.3)	5.8(5.6)	5.3(4.3)	3.1(2.6)
Syracuse, NY	6.1	5.7(4.2)	3.9(3.6)	3.6(2.4)	3.5(2.8)
Lansing, MI	4.5	4.5(1.5)	3.7(3.1)	2.9(2.7)	2.5(1.7)
Madison, WI	4.7	4.7(3.9)	3.2(1.9)	2.8(1.7)	2.4(2.0)
Marquette, MI	8.5	8.5(6.0)	8.4(7.2)	8.1(6.4)	7.3(4.3)
Milwaukee, WI	5.5	5.5(3.2)	3.1(2.3)	2.5(2.1)	2.5(2.1)
Muskegon, MI	6.5	4.6(3.2)	3.9(3.0)	3.5(2.7)	3.1(2.3)
Green Bay, WI	4.7	4.7(3.0)	4.3(3.4)	3.9(3.6)	2.3(1.5)
Sault Ste. Marie, MI	13.0	13.0(12.5)	12.2(8.7)	11.4(6.7)	9.9(9.6)
South Bend, IN	7.2	7.2(4.5)	5.1(4.0)		
Erie, PA	3.8	1.9(1.8)	1.9(1.8)		
Duluth, MN	10.6	10.6(8.3)	7.5(3.3)	6.9(5.4)	6.5(5.5)
International Falls, MN	8.3	8.3(3.5)	5.2(4.6)	4.4(3.4)	2.6(4.0)
La Crosse, WI	4.4	4.4(4.2)	3.0(2.0)	2.8(1.5)	2.6(1.5)
Minneapolis, MN	6.6	6.6(5.9)	6.3(5.1)	5.0(4.6)	3.9(3.2)
Rochester, MN	4.9	4.9(3.9)	4.1(1.5)	4.0(1.4)	3.0(2.9)
St. Cloud, MN	7.7	7.7(6.2)	4.6(4.5)	4.4(4.3)	3.8(3.1)
Omaha, NE	4.4	3.0(1.4)	2.1(1.5)	2.0(0.6)	1.9(1.3)
Sioux City, IA	5.4	4.5(1.6)	3.9(3.8)	2.5(1.0)	
Worcester, MA	5.5	5.5(1.6)	4.6(2.7)	3.8(2.3)	3.3(1.9)
Houghton Lake, MI	5.4	4.9(3.7)	4.2(2.4)	3.1(2.0)	3.0(2.1)
Pittsburgh, PA	3.6	2.8(2.1)	2.3(0.6)	2.1(1.5)	
Alpena, MI	6.5	6.5(6.1)	6.5(5.8)	5.2(2.8)	4.5(2.9)
Grand Rapids, MI	4.3	3.6(2.4)	3.0(2.2)	3.0(2.0)	2.7(1.1)
Dubuque, IA	6.2	5.3(4.3)	4.2(3.6)	2.7(2.1)	2.5(2.2)
Waterloo, IA	4.8	4.8(2.2)	3.5(2.5)	3.3(1.7)	2.4(2.0)

NOTE: The years are not included.

W_x set is generally 1.5 to 2 times larger than the water equivalent associated with the S_x set of data. A similar relation exists between densities.

Based on the above observations and statistical studies on the two sets of data it was concluded that regression relationships should be developed on the set of annual maximum water equivalents, the W_x set. These relationships will be applied to the cooperative weather network data by making an estimate for each day with snow on the ground and then selecting the extreme estimated water equivalent after verifying the physical consistency of the cooperative data.

c. Analysis of Predictor Variables

Although many variables were considered, only those exhibiting definite promise as predictor variables are discussed. Figures 6 through 10 show the correlation fields of the most promising predictors with the natural logarithm of water equivalent.

The natural logarithms of cumulative precipitation (LMM) and snow depth (S) are the two variables most highly related to water equivalent. The number of days in the string with continuous snow on the ground (NDAYS) prior to the observed annual maximum water equivalent, the number of times daily observations of snow depth increase by at least one inch (LYR), the number of times that snow depth decreases by at least one inch (KRST) prior to the date of annual maximum water equivalent and the number of snowfalls greater than one inch (NSNO) are all positively correlated to the natural logarithm of annual maximum water equivalent and are considered as the potential predictors to be used in this study. Other variables added little information to predictive equations.

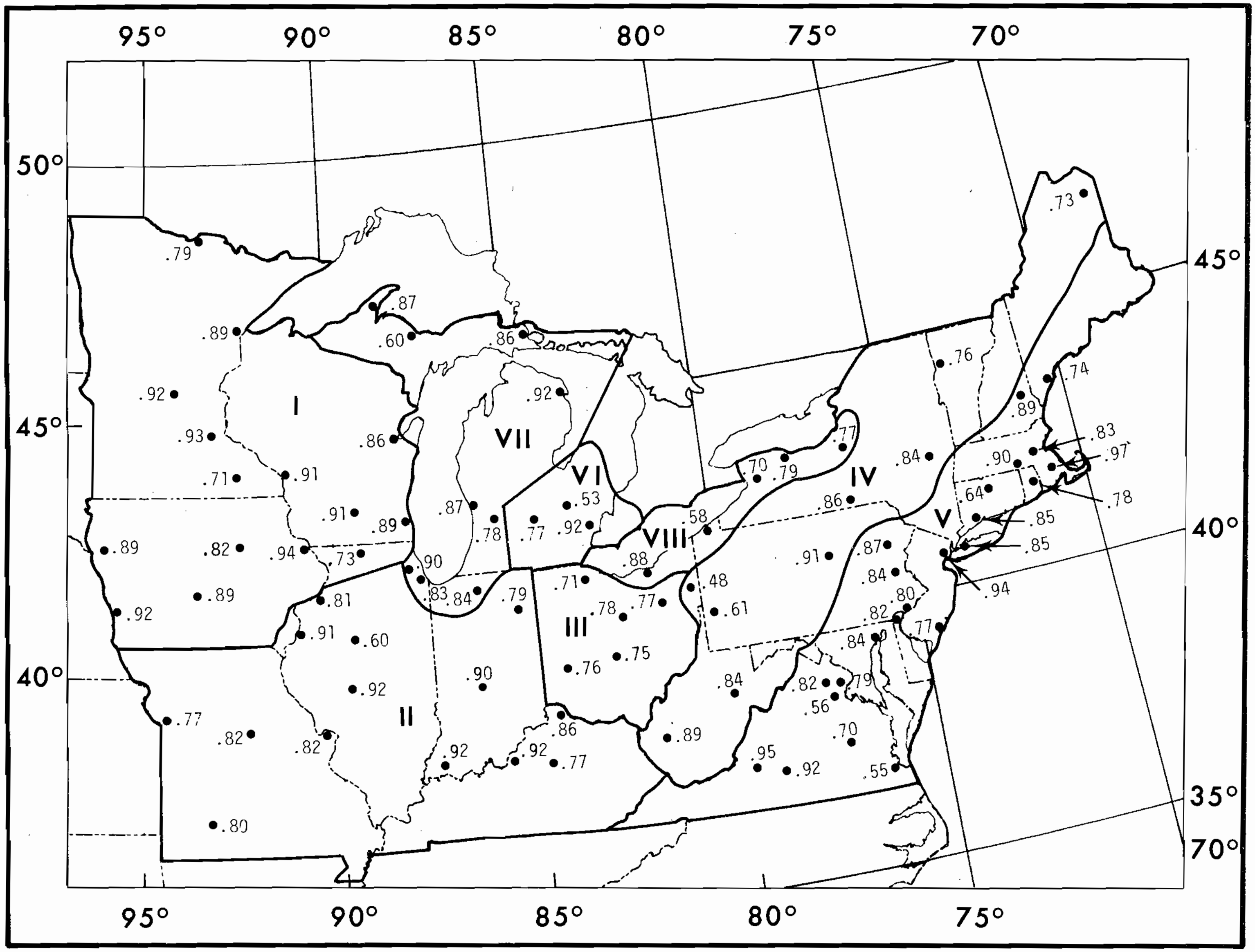

Figure 6. Correlation of $\log_e$ of annual maximum water equivalent and the $\log_e$ of cumulative precipitation (LMM) from the sample of annual water equivalent. Negative or insignificant correlations (at α = .05), or correlations with very small sample sizes are not included.

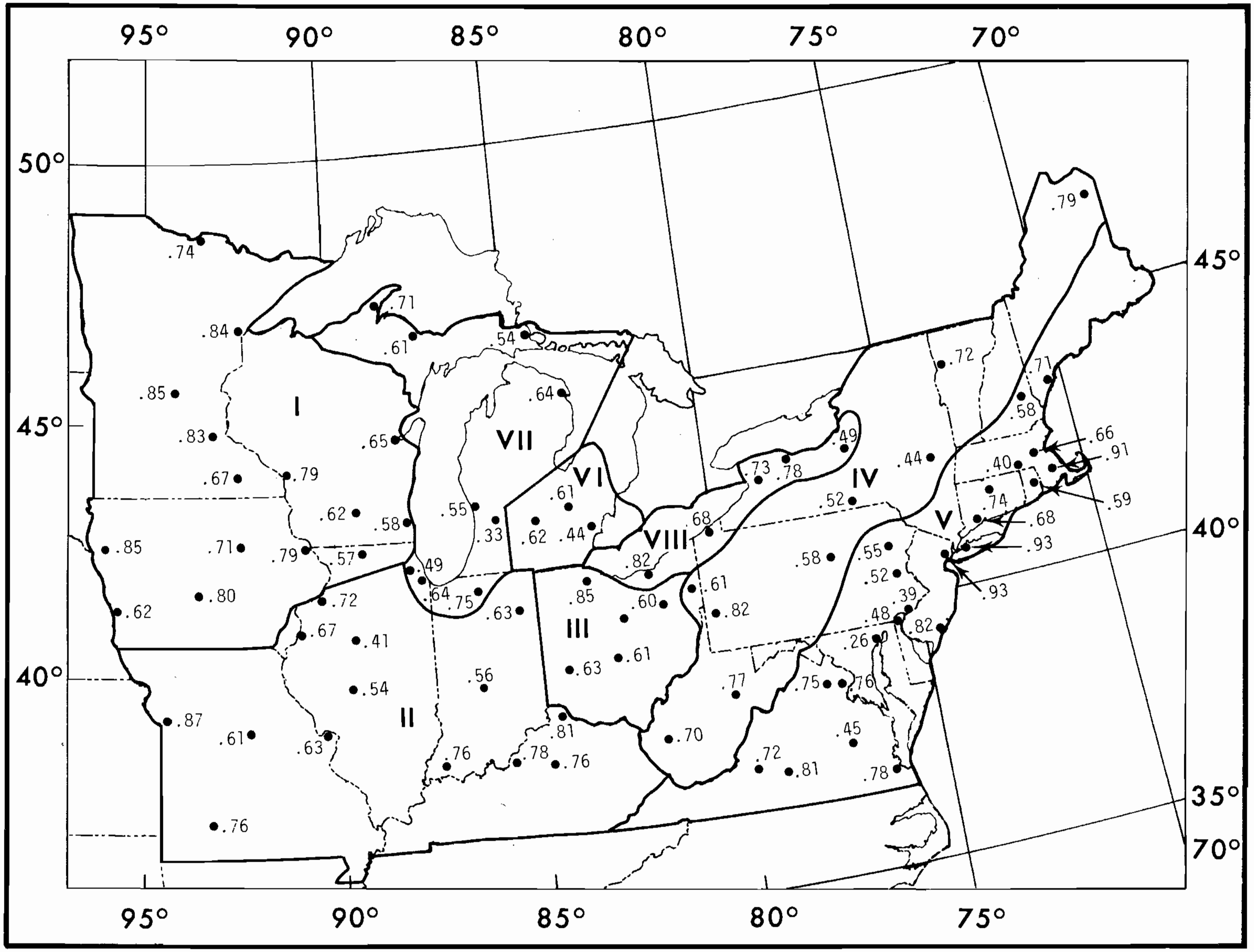

Figure 7. Correlations of $\log_e$ of annual maximum water equivalent and the $\log_e$ snow depth (S) from the sample of annual maximum water equivalent. Negative or insignificant correlations (at α = .05), or correlations with very small sample sizes are not included.

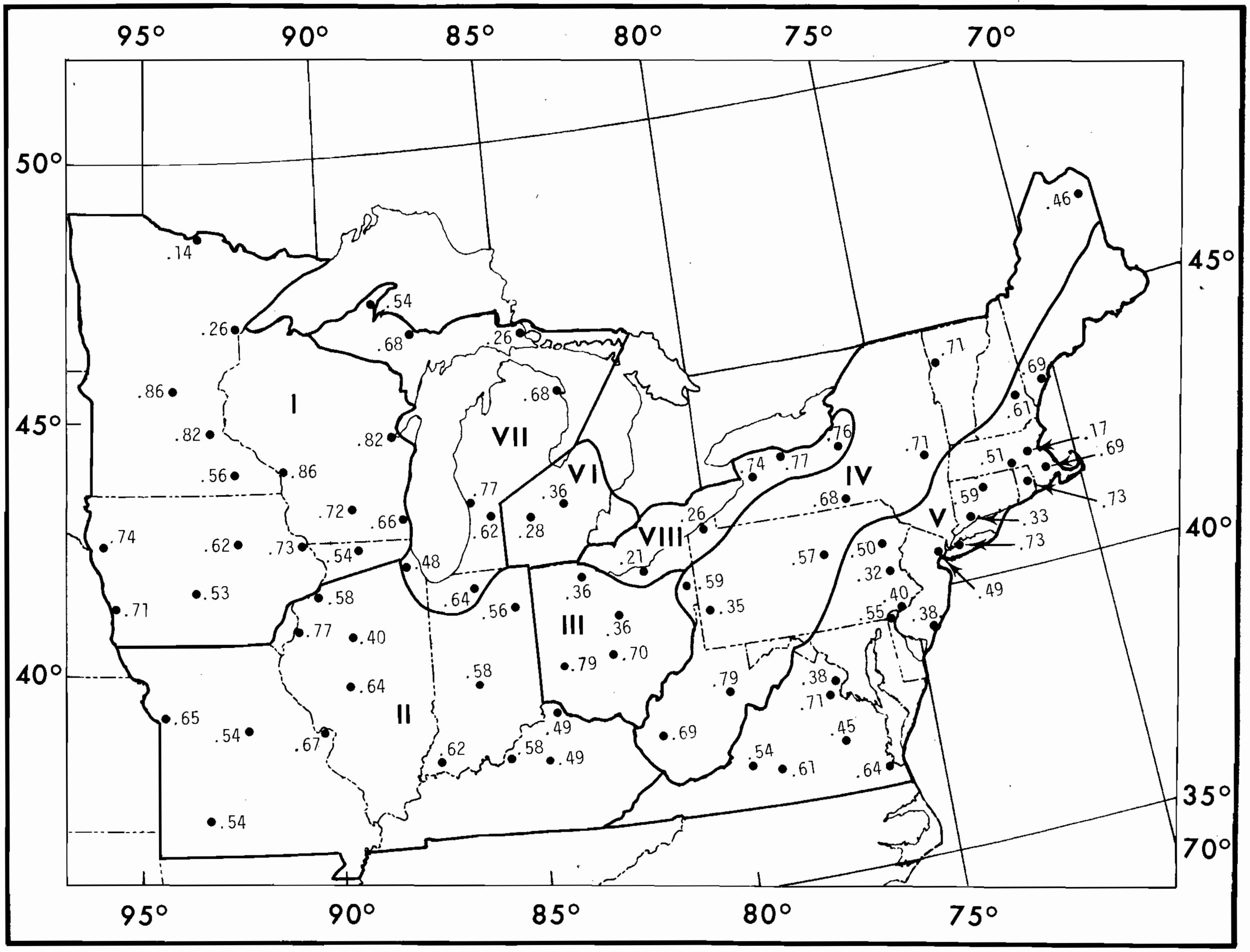

Figure 8. Correlation of $\log_e$ of annual maximum water equivalent and the number of layers of snow (LYR) from the sample of annual maximum water equivalent. Negative or insignificant correlations, (at α = .05) or correlations with very small sample sizes are not included.

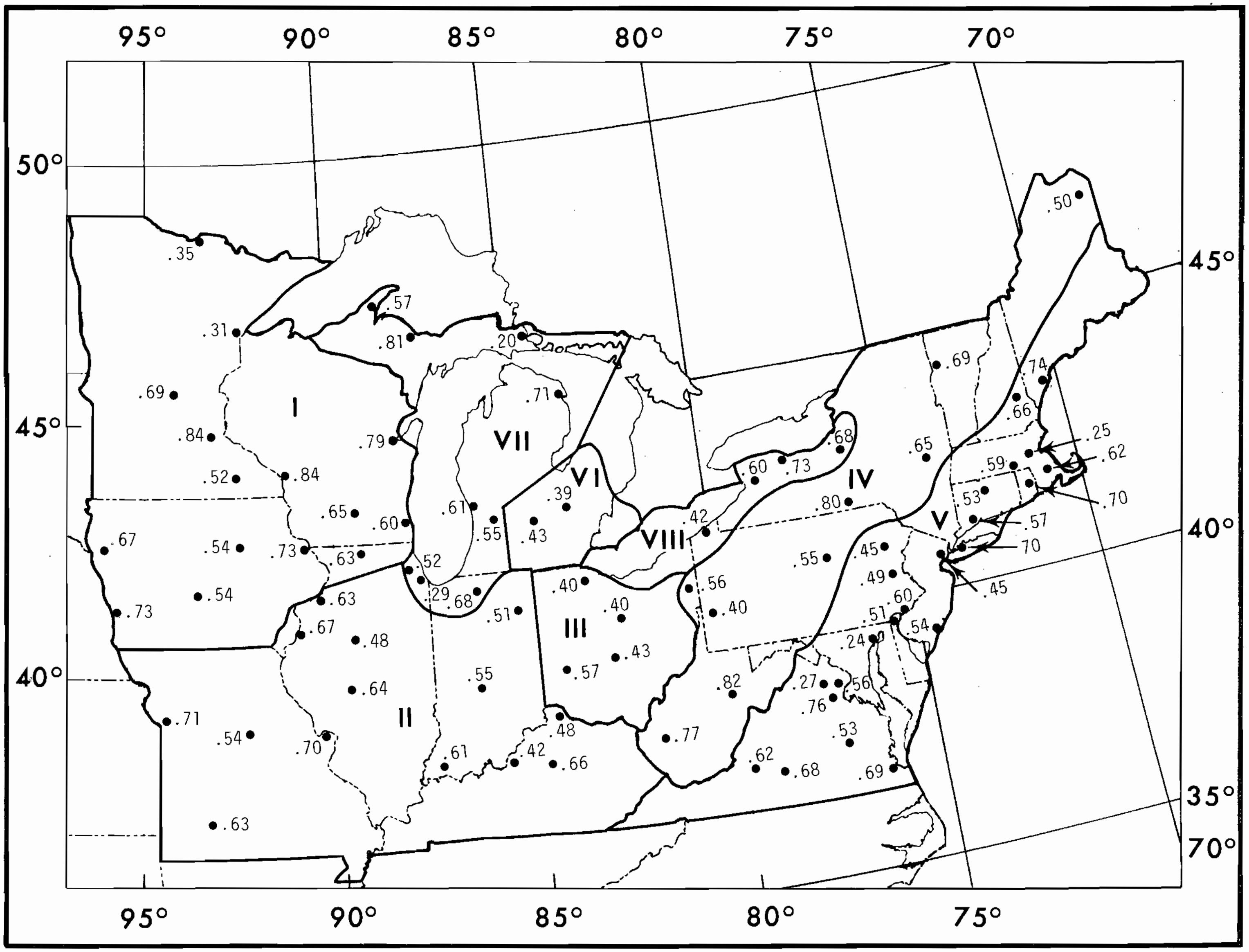

Figure 9. Correlations of $\log_e$ of annual maximum water equivalent and number of crusted layers (KRST) from the sample of annual maximum water equivalent. Negative or insignificant correlations, (at α = .05) or correlations with very small sample sizes are not included.

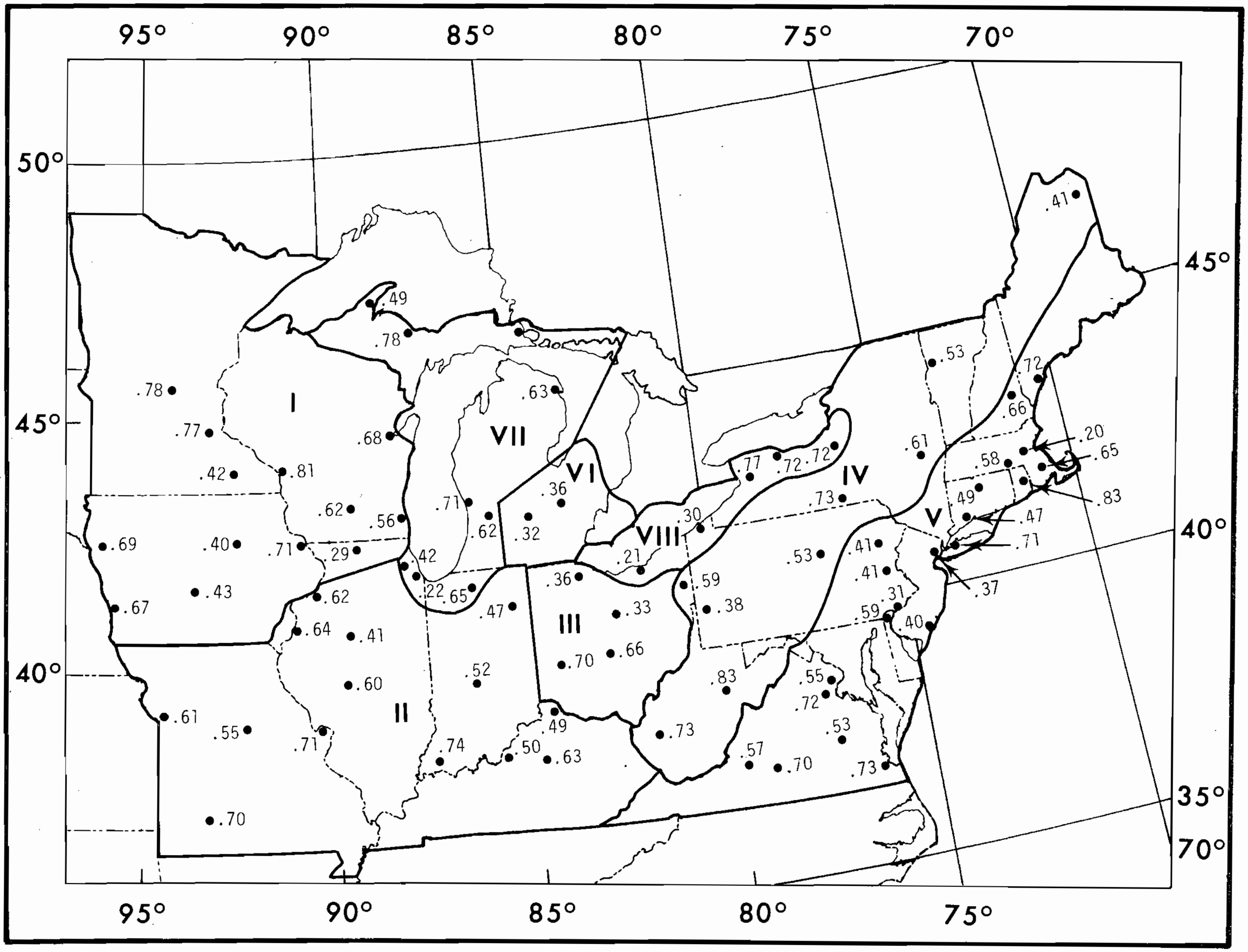

Figure 10. Correlations of $\log_e$ of annual maximum water equivalent and the number of days with continuous snow on the ground (NDAYS) from the sample of annual maximum water equivalent. Negative or insignificant correlations, (at α = .05) or correlations with very small sample sizes are not included.

The variable KRST is analogous to the number of crusted layers and is a composite variable which reflects the effects of temperature, wind movement, settling, evaporation, freezing, thawing and other effects which may not be directly available from cooperative observing stations.

d. Models and Independent Tests

Based on the individual correlations and results of stepwise regression analysis for each station, it was generally concluded that the natural logarithm of cumulative precipitation was the most significant predictor of water equivalent followed by the natural logarithm of snow depth. Therefore, the predictors LMM and S appear in all models with other terms being included if they contributed significantly to the regression equations. Tables 2 and 3 show, by region, the variables, explained variance and standard errors of the linear and non-linear regression analysis results. The standard errors given in Table 3 are in inches and are additive, since the non-linear models were developed on actual water equivalents and not on the natural logarithm of water equivalents. The standard errors in Table 2 are for the logarithm of the water equivalent. This means that the standard errors are multiplicative.

For example, the model for region 7 is:

$$\hat{u} = -.55 + .66LMM + .30S + .006NSNO$$

This model has an explained variance of 88 percent and a standard error of .30. (Note that for a $\hat{u} = .92$ or 2.5 inches of estimated water equivalent, this standard error indicates a range on the estimate of $2.5e^{-.30}$ to $2.5e^{+.30}$ or 1.85 inches to 3.37 inches).

Table 2

Terms, Explained Variance, and Standard Error by Region for Linear Regression Models Developed on W_x Set of Data

Region	Variables	Explained Variance	Log_e Standard Error (in)*
1	LMM, S, KRST	85%	0.30
2	LMM, S, NDAYS	74%	0.27
3	LMM, S	65%	0.33
4	LMM, S, NSNO	82%	0.33
5	LMM, S, KRST	83%	0.30
6	LMM, S	64%	0.38
7	LMM, S, NSNO	88%	0.30
8	LMM, S, KRST	72%	0.36

*Standard Error is multiplicative because of the natural logarithm transformation.

Table 3

Non-Linear Regression Analysis

Region	Variables	Explained Variance	Standard Error (in)
1	LMM, S, NSNO	94%	0.70
2	LMM, S, NSNO	92%	0.30
3	LMM, S	91%	0.26
4	LMM, S	91%	0.89
5	LMM, S, NSNO	93%	0.54
6	LMM, S	91%	0.57
7	LMM, S	94%	0.93
8	LMM, S, NSNO	89%	0.84

Independent test results on the winters of 1977/1978 and 1978/1979 data indicate that the non-linear models tend to predict slightly closer to the actual maximum water equivalent than do the linear models for all regions except region 2. Based on independent tests and dependent statistics derived from the data samples used in developing the models for the period 1952-76, it is concluded that non-linear regression models are superior to the other types of linear regression models which were developed and tested as part of this study, except for region 2.

The predictions made from Swedish and Norwegian bulk density type models (6) were compared to those made from non-linear models. The foreign models predicted a mean bulk density that is used with snow depth to estimate water equivalent. The results of the comparative analysis are very important because they suggest a possible improvement in design of the non-linear models. These foreign density models were determined to be superior to the non-linear models when predicting extremes for all regions using 1977/78 and 1978/79 data. These results are preliminary. Rigorous meteorological and statistical analysis of the results is required. Based on observations from the 1977/78 and 1978/79 test results, it is concluded that:

(1) It is possible that regression relationships could be used with better results if the models were developed with bulk density instead of water equivalent. (Note that the density data still require observed water equivalents.)

(2) A more logical reason for these results is that in the sample used to develop the non-linear models, the mean water equivalent is sufficiently small in magnitude to restrict the model's ability to predict upper extremes. Regression equations make the best predictions near the mean of the sample. The maximum values in the test data are in some cases 2 to 4 times larger than the mean sample values. Considering the number of cases in each region, a stratification to obtain a higher mean sample water equivalent is feasible.

(3) Considering the large mean snow depth conditions in Scandinavia, the density models are actually predicting on United States data which are low by Scandinavian standards. Therefore, it is likely that mean sample conditions, not model design, are responsible for these results.

B. Phase II - Cooperative Station Data Analysis

1. Methods of Analysis

a. Data Derivation

Based on the results of Phase I, daily water equivalent estimates were derived from observed daily data for all cooperative and first order stations. The non-linear (linear in Region 2), Swedish and Norwegian models given in Appendix A were separately applied to the daily data. Therefore, three estimates of water equivalent were computed for each station-year.

Results of independent tests in Phase I seem to indicate that the two Scandinavian models were as good as or possibly better than the linear and non-linear models when applied to water equivalent data in the upper extremes of the distributions. The linear and non-linear models appeared to be as good as or slightly better than the Scandinavian models when applied to data which is closer to the central portion of the distributions. Therefore, in order to "build" some conservatism (high estimates) into the data sets, the annual maximum water equivalent estimate was taken as the highest value for each year of the three values generated from the non-linear (linear in Region 2), Swedish and Norwegian models.

information). Figure 11 is a sample plot of the Fisher-Tippett, Type I model applied to the water equivalent data for Danbury, CT.

Because the K-S test is a more powerful test than the Chi Square test (13), it was concluded that either the Weibull or the Fisher-Tippett, Type I models should be used to derive water equivalent estimates for various probability levels. Examination of the estimates indicate that there is very little difference between the two models. The Fisher-Tippett, Type I model was chosen over the Weibull model to graphically present the spatial distribution of annual extreme snow load in pounds per square foot because the necessary computer software was already available.

2. Model Application

a. Fisher-Tippett, Type I Model

Analysis of extreme value data is best accomplished by fitting an extreme value distribution to the data. A set of extreme data of sample size N, assuming the Fisher-Tippett, Type I extreme value distribution, can be described by the cumulative probability density function

$$F(x) = F(x;\alpha,\beta) = \exp\{-\exp[-(x-\alpha)/\beta]\} = \exp[-\exp(-y)] \qquad (3)$$

where $F(x)$ is the probability that an observation will be less than or equal to a specified value x, α is the mode of the distribution, β is the scale parameter and y is a reduced variate analogous to the standardized variate of the normal distribution. The reduced variate is given by

$$y = (x-\alpha)/\beta \qquad (4)$$

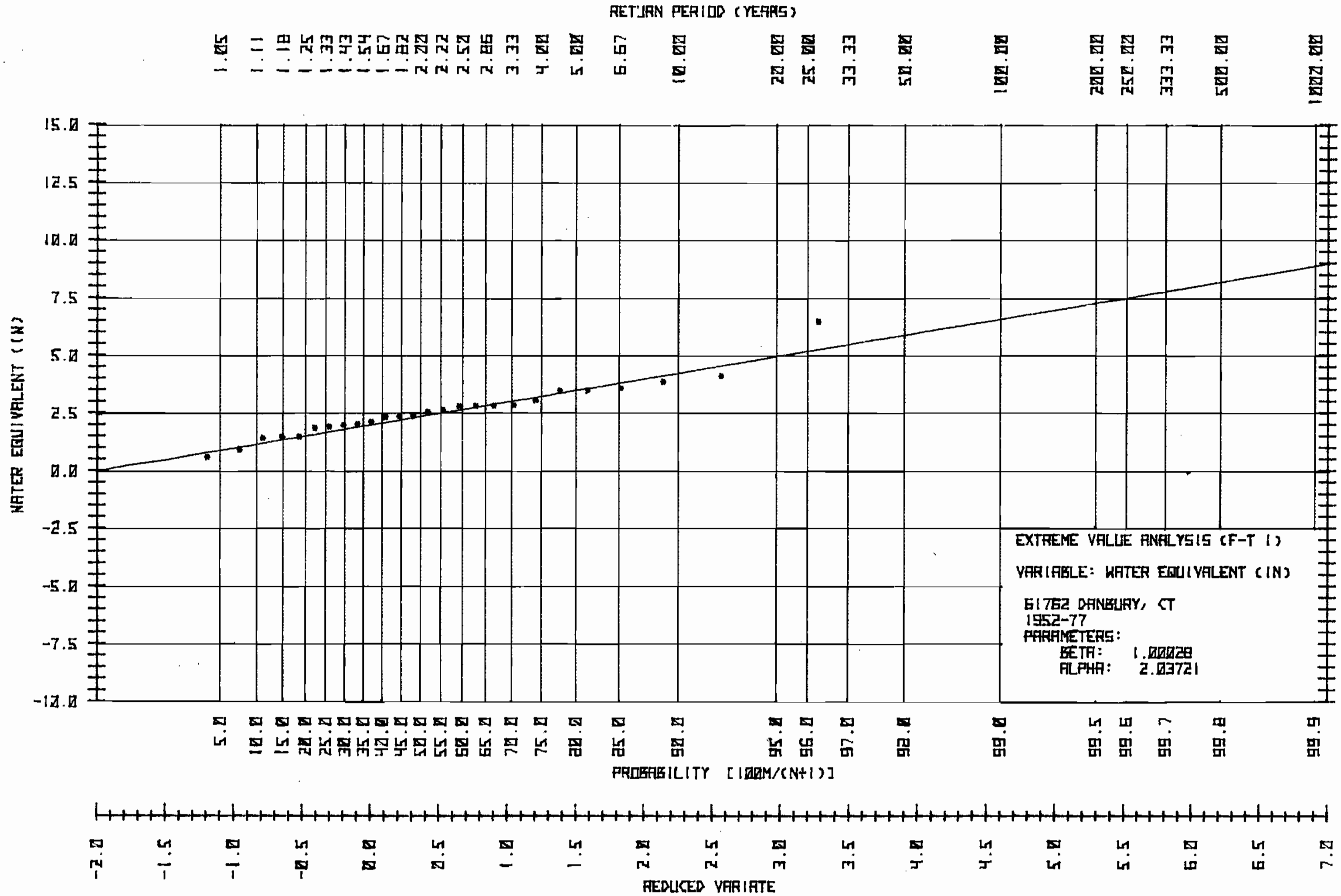

Figure 11. Graphical extreme value analysis of annual extreme water equivalents for Danbury, CT for the winter seasons 1952-53 through 1977-78.

This distribution has been extensively investigated by Gumbel (3,4) and Lieblein (10,11,12) and applied to meteorological extremes by Thom (14,15).

Estimation and prediction of extreme values for a given probability can be accomplished from the relationship

$$\xi_p = \alpha + y_p\beta \tag{5}$$

where ξ_p is the extreme value at probability level p and y_p is the corresponding reduced variate. The estimated extreme value at a probability level p, $\hat{\xi}_p$, from sample data is determined from estimates $\hat{\alpha}$, $\hat{\beta}$ of the distribution parameters

$$\hat{\xi}_p = \hat{\alpha} + y_p\hat{\beta} \tag{6}$$

Estimated extreme values $\hat{\xi}_p$ in this study were based on the best linear unbiased estimates $\hat{\alpha}$, $\hat{\beta}$ computed from the order statistics estimating approach of Lieblein (10,11,12).

The standard deviation of the estimator $\hat{\xi}_p$ is a measure of the reliability or confidence of the estimator, i.e., the extent to which repeated applications of the procedures to repeated samples taken under the same conditions would give values clustering around the unknown parameter value. Although tables of the standard deviation of $\hat{\xi}_p$ for sample sizes used in this study are not available, it is inferred from Lieblein (10) that the standard deviation is inversely proportional to the square root of the sample size. For example, quadrupling the sample size will halve the standard deviation. Confidence intervals around ξ_p can be constructed if the distribution of $\hat{\xi}_p$ in repeated sampling and the standard deviation of $\hat{\xi}_p$ are known.

b. Results

The above model was applied to estimated annual extreme water equivalent data sets for approximately 300 cooperative stations. Estimated water equivalent values were derived for the .5, .98 and .99 probability levels corresponding to the 2-, 50-, and 100-year return periods. These estimates were converted to horizontal snow loads (in pounds per square foot), plotted and analyzed. Figures 12, 13 and 14 show the results of this snow load analysis for the northeastern quarter of the United States.

Certain considerations should be noted when using these maps to interpolate snow load estimates for the given probability levels or with the intent of deriving estimates for other probability levels.

(1) Interpolation to points between major isolines of snow load is risky because there is not, in general, a linear gradation between isolines. Linearly interpolated values to intermediate points can be considerably different from actual computed values.

(2) Mountainous regions indicated by the dotted pattern sometimes exhibit extreme lateral variation in snow load estimates due to local topographic influences. Interpolations in these regions can be misleading. If possible, individual station analyses (computations) should be performed.

(3) Probability estimates in this study are considered to be slightly conservative (high) with respect to the actual water equivalent results. Linear, non-linear and the Scandinavian models were used to generate estimates of the annual extreme water equivalent for approximately 1100 cooperative and first order stations.

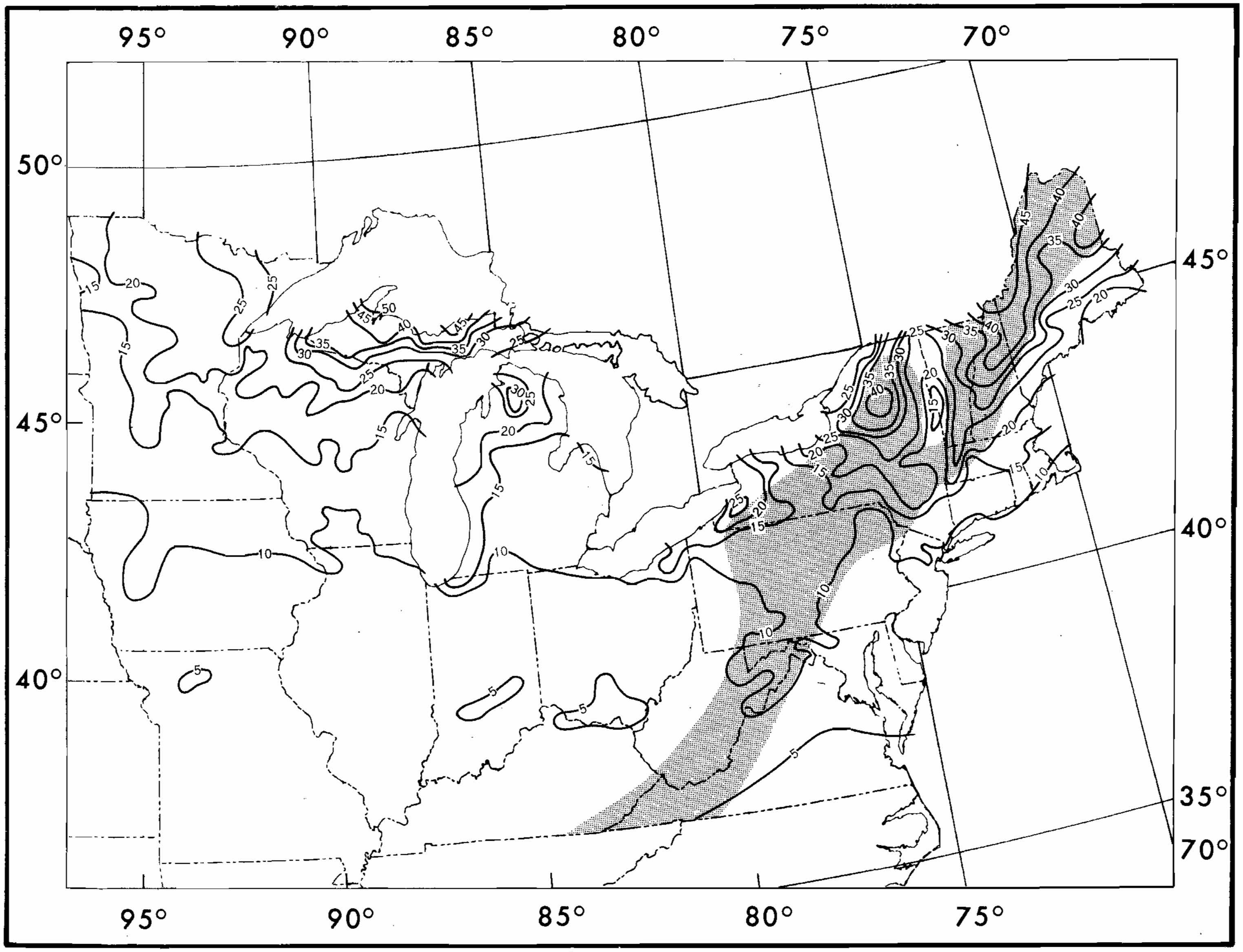

Figure 12. Snow load distribution (lbs./ft.^{2}) for the .5 probability level (2-year return period). The dotted area indicates regions where local topography may dictate considerable differences in horizontal extent of snow load estimates. Individual station analyses are recommended for this area.

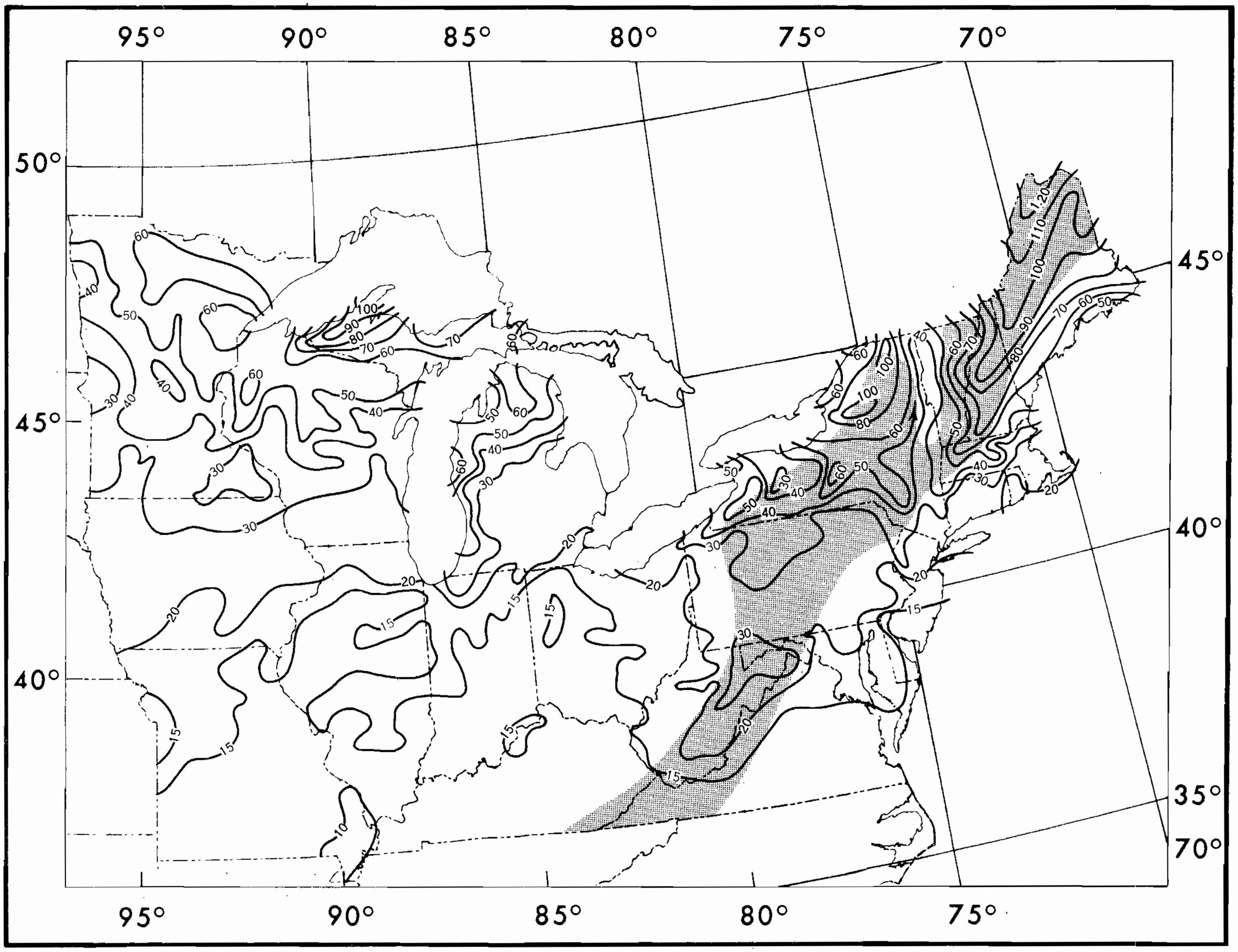

Figure 13. Snow load distribution (lbs./ft.^{2}) for the .98 probability level (50-year return period). The dotted area indicates regions where local topography may dictate considerable differences in horizontal extent of snow load estimates. Individual station analyses are recommended for this area.

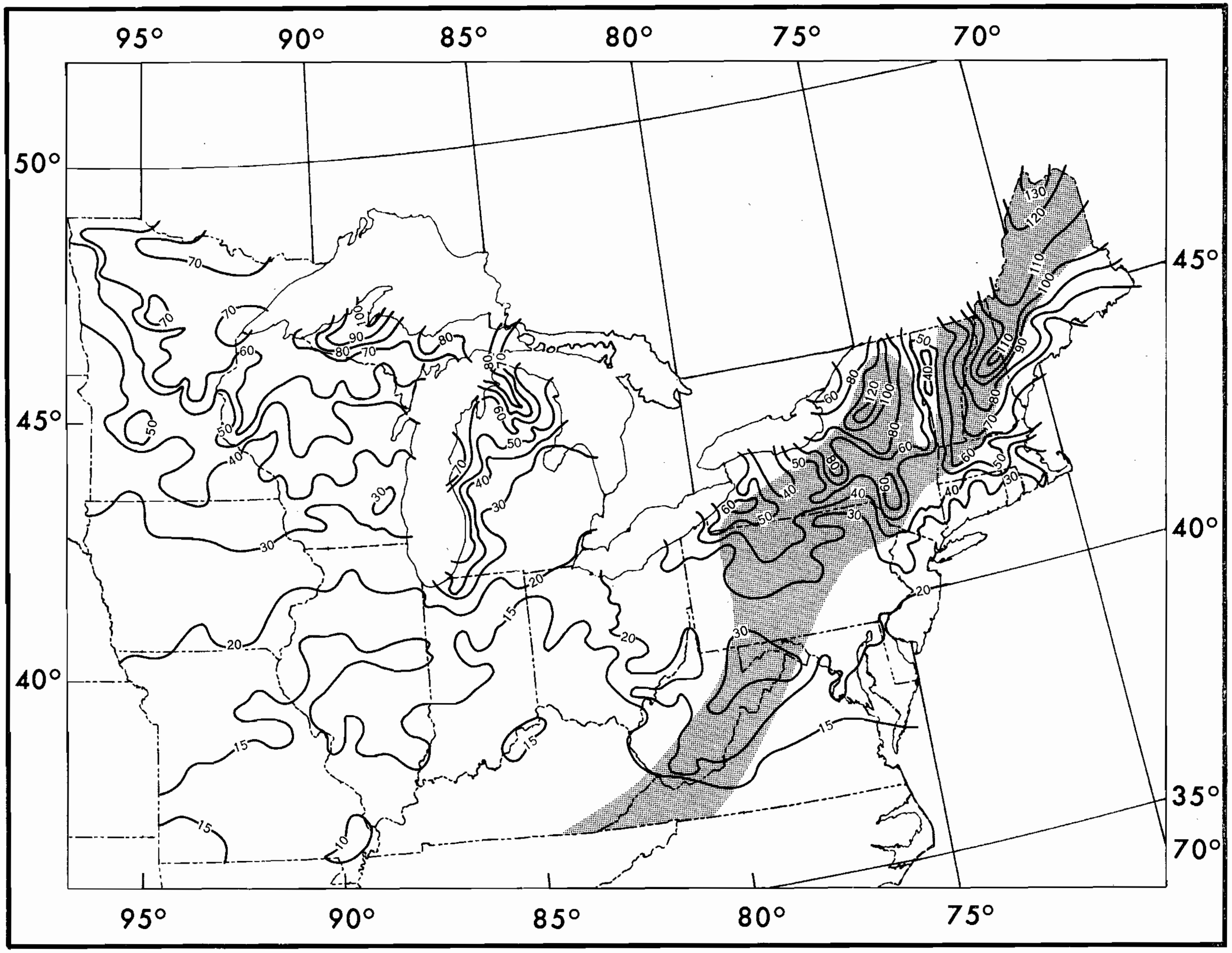

Figure 14. Snow load distribution (lbs./ft.^{2}) for the .99 probability level (100-year return period). The dotted area indicates regions where local topography may dictate considerable differences in horizontal extent of snow load estimates. Individual station analyses are recommended for this area.

Comparisons were made between the probability estimates obtained from the actual water equivalent series from first order stations with the probability estimates computed from the "estimated" series. In nearly all cases the probability estimates obtained from the actual data series were lower than those estimates derived from the "estimated" data series (see Table 6).

IV. RECOMMENDATIONS AND CONCLUSIONS

From the results of this study it is concluded that in general non-linear regression models tend to predict water equivalent values better than linear regression models. This is particularly true in cases where the actual water equivalent value and its associated predictors are near the central portion of the distribution. However, preliminary results show that extreme water equivalent values seem to be predicted more accurately by two Scandinavian bulk density models.

A "synthetic" water equivalent data base for cooperative and first order stations was developed from which probability estimates of water equivalent (snow load) could be made. The decision to use the maximum of the model estimates produced a data base which is considered by the authors to be slightly conservative (high). No other methods of conservatism are incorporated into this study.

Map analyses of snow load on the ground are presented from which probability estimates can be obtained. These estimates are the results of repeated application of the Fisher-Tippett, Type I extreme value model to the derived data base for each station. It is noted that in the mountainous areas of the northeastern United States spatial distribution of snow loads is highly

Table 6

The frequency and percentage of occurrences that the difference in the 50-year return period value (Estimated Value - Actual Value) for selected first order stations was within the specified ranges.

	Ranges (lbs/Ft^2)									
	<-15	-15 to -10	-10 to -5	-5 to 0	0 to 5	5 to 10	10 to 15	15 to 20	20 to 25	≥ 25
Frequency	0	1	0	13	32	12	5	2	1	0
Percent	0	1.5	0	19.7	48.5	18.2	7.6	3.0	1.5	0

variable. Individual station analyses should be performed in these areas where possible.

It is recommended that the following areas be investigated and performed:

1. Detailed specifications be developed in the confidence bands theory of these snow load estimates. This would enable some statements to be made on reliability of the data.

2. Develop a data base for other stations in the United States outside this area of study.

3. Additional studies should be undertaken to develop appropriate method for deriving estimates for locations in mountainous regions.

4. Additional evaluation of the bulk density models should be made with the idea of a possible improvement in predictive capabilities.

Acknowledgements

The authors wish to acknowledge the excellent graphic preparation done by Mr. Robert Courtney and the assistance of the Department of Atmospheric Science, University of Missouri in data preparation. This work was sponsored by The Nuclear Regulatory Commission under Interagency Agreement Number NRC-01-77-109.

Bibliography

1. Falls, L. W., 1971: "A Computer Program for Standard Statistical Distributions", NASA Technical Memorandum, NASA TM X-64588, George C. Marshall Space Flight Center, Huntsville, Ala., pp 86.

2. Goodison, B. E. and D. J. McKay, 1978: "Canadian Snowfall Measurements: Some Implications for the Collection and Analysis of Data from Remote Stations." Paper presented at the Western Snow Conference, Otter Rock, Oregon, April 18-20, 1978, 9 pp.

3. Gumbel, E.J., 1958: Statistical of Extremes, Columbia University Press, pp 375.

4. Gumbel, E.J., and J. Lieblein, 1954: "Some Applications of Extreme Value Methods", American Statistician, 8:14.

5. Hendrick, Robert L. and Roger J. DeAngelis, 1976: "Seasonal Snow Accumulation, Melt and Water Input--A New England Model." Journal of Applied Meteorology, 15:7:717-727.

6. International Organization for Standardization, 1975: Fifth draft proposal, including the improvements and additions to the draft proposal approved at SC 3 meeting 1975-10-6/07/08, Norges Byggstandardiseringsrad, Building Division of NSF, Kobenhavngt. 10, OSLO 5.

7. Kovacs, G. and G. Molnar, 1974: "Determination of Snow Water Equivalent and Snow Melt Water by Thickness Snow Cover Data." Hydrological Sciences-Bulletin-des Sciences Hydrologiques, XIX:435-445.

8. Larson, Lee W. and Eugene L. Peck, 1974: "Accuracy of Precipitation Measurements for Hydrologic Modeling." Water Resources Research, 10:4:857-863.

9. Lemmela, R. and E. Kusisto, 1974: "Evaporation from Snow Cover." Hydrological Sciences-Bulletin-des Sciences Hydrologiques, XIX:541-548.

10. Lieblein, J., 1954: "A New Method of Analyzing Extreme Value Data", Technical Note 3053, National Advisory Committee for Aeronautics, pp 88.

11. Lieblein, J., 1974: "Efficient Methods of Extreme-Value Methodology", NBSIR 74-602, U.S. Department of Commerce, National Bureau of Standards, pp 32.

12. Lieblein, J., 1974: "Note on Simplified Estimators for Type I Extreme-Value Distribution", NBSIR 75-637, U.S. Department of Commerce, National Bureau of Standards, pp 12.

13. Siegel, S., 1956: Nonparametric Statistics for the Behavioral Sciences, McGraw-Hill Book Company, Inc., pp 127-136.

14. Thom, H.C.S., 1966: "Distribution of Maximum Annual Water Equivalent of Snow on the Ground." Monthly Weather Review, 94:265-271.

15. American National Standards Institute (ANSI), 1972, A58.1-1972, "Building Code Regulations for Minimum Design Load in Buildings and Other Structures," Washington, D.C., figures 3 and 4.

16. Tobiasson W. and R. Redfield, 1973: "Alaskan Snow Loads." Paper presented at the 24th Alaska Science Conference, University of Alaska, August 1973, 42 pp.

17. Tobiasson, W. and R. Redfield, 1977: "Update on Snow Load Research at CRREL." Paper presented at the 34th Eastern Snow Conference, Belleville, Ontario, February 3-4, 1977, 6 pp.

18. United States Weather Bureau, 1964: "Frequency of Maximum Water Equivalent of March Snow Cover in North Central United States." Technical Paper No. 50, Washington, D.C., 24 pp.

Appendix A

Predictor Equations by Region:

Region	
1	$.574 \exp(.37\ S) \exp(.008\ NSNO) \exp(.59\ LMM)$
2	$-.91 + .52\ LMM + .39\ S + .016\ NDAYS$
3	$.29 \exp(.54\ S) \exp(.62\ LMM)$
4	$.36 \exp(.55\ S) \exp(.55\ LMM)$
5	$.6 \exp(.25\ S) \exp(.011\ NSNO) \exp(.64\ LMM)$
6	$.34 \exp(.68\ S) \exp(.53\ LMM)$
7	$.54 \exp(.36\ S) \exp(.69\ LMM)$
8	$.65 \exp(.3\ S) \exp(.016\ NSNO) \exp(.4\ LMM)$

where:

LMM is the log of the sum of precipitation (inches)

S is the log of snow depth (inches)

NSNO is the number of snowfalls

NDAYS is the number of days snow has been on the ground.

Bulk Density Models

$\hat{\rho} = 300.0 - 200.0 \exp(-1.5\ d)$, Norway

$\hat{\rho} = 155.0 + 0.7\ t$, Sweden

where:

$\hat{\rho}$ is the estimated snow density (Kg/m^3)

t is the duration of snow on the ground after November 1 (in days)

d is the snow depth (meters)

NRC FORM 335 (7-77)

U.S. NUCLEAR REGULATORY COMMISSION

BIBLIOGRAPHIC DATA SHEET

1. REPORT NUMBER (Assigned by DDC)
NUREG/CR-1389

4. TITLE AND SUBTITLE (Add Volume No., if appropriate)
ESTIMATING WATER EQUIVALENT SNOW DEPTH FROM RELATED METEOROLOGICAL VARIABLES

2. (Leave blank)

3. RECIPIENT'S ACCESSION NO.

7. AUTHOR(S) Louis T. Steyaert, Sharon K. Le Duc, Norton D. Strommen, M. Lawrence Nicodemus and Nathaniel B. Guttman

5. DATE REPORT COMPLETED

MONTH	YEAR
February	1980

9. PERFORMING ORGANIZATION NAME AND MAILING ADDRESS (Include Zip Code)
National Climatic Center
Environmental Data Service
National Oceanic and Atmospheric Administration
Federal Building, Asheville, North Carolina 28801

DATE REPORT ISSUED

MONTH	YEAR
May	1980

6. (Leave blank)

8. (Leave blank)

12. SPONSORING ORGANIZATION NAME AND MAILING ADDRESS (Include Zip Code)
Office of Standards Development
U. S. Nuclear Regulatory Commission
Washington, DC 20555

10. PROJECT/TASK/WORK UNIT NO.
SS-001-7

11. CONTRACT NO.
FIN No. B1080

13. TYPE OF REPORT
NUREG

PERIOD COVERED (Inclusive dates)

15. SUPPLEMENTARY NOTES

14. (Leave blank)

16. ABSTRACT (200 words or less)
Engineering design must take into consideration natural loads and stresses caused by meteorological elements, such as, wind, snow, precipitation and temperature. The purpose of this study was to determine a relationship of water equivalent snow depth measurements to meteorological variables. Several predictor models were evaluated for use in estimating water equivalent values. These models include linear regression, principal component regression, and non-linear regression models. Linear, non-linear and Scandanavian models are used to generate annual water equivalent estimates for approximately 1100 cooperative data stations where predictor variables are available, but which have no water equivalent measurements. These estimates are used to develop probability estimates of snow load for each station. Map analyses for 3 probability levels are presented.

17. KEY WORDS AND DOCUMENT ANALYSIS

17a. DESCRIPTORS

17b. IDENTIFIERS/OPEN-ENDED TERMS

18. AVAILABILITY STATEMENT

19. SECURITY CLASS (This report)
Unclassified

20. SECURITY CLASS (This page)
Unclassified

21. NO. OF PAGES

22. PRICE
$

NRC FORM 335 (7-77)

UNITED STATES
NUCLEAR REGULATORY COMMISSION
WASHINGTON, D. C. 20555

OFFICIAL BUSINESS
PENALTY FOR PRIVATE USE, $300

POSTAGE AND FEES PAID
U.S. NUCLEAR REGULATORY
COMMISSION

U.S. MAIL