WIND AID OR EQUIVALENT WIND
CALCULATION BY SAWYER'S[1] METHOD

by

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1. Introduction. Wind aid (equivalent wind) may be defined as the aid or retard an airborne craft receives as it courses from one point to another within our atmospheric shell. If the craft must follow a prescribed route, this aid, positive or negative, assumes a major role in operational planning.

2. Distribution of Winds. The assumption of the normality of distribution of winds has been made many times. This assumption is that the vector mean wind is an average wind for the period and that all of the individually observed winds deviating from this mean are distributed in the normal sense about the end of the vector mean. The standard vector deviation is the magnitude of the radius of a circle which encompasses 63% of the deviations and, therefore, 63% of the end points of the individually observed winds.

Various radii can be used which will enclose certain percentages. These radii can be determined by the use of the following equation:

$$r = \sigma_r \left(\ln_e \frac{1}{1-p}\right)^{1/2} \qquad (1)^2$$

where

- r is the radius
- σ_r is the magnitude of the Standard Vector Deviation; it has no direction,
- p is the percentage of observations enclosed by the circle of radius r.

Now, the distribution of the components of the deviations along a prescribed ground course are normally distributed in the linear sense, not the circular. The magnitude of the radius of circles which intercept specified percentages of the deviations along the flight path (aid) may be determined from Table 1. These centers will be concentric about the value of the mean wind aid. Graphical methods for the determination of wind aid were presented in a paper by the writer before the American Meteorological Society in Miami, Florida, November 18, 1954[3].

Table 1. Radii of Circles which intercept Specified Portions of Distributed Wind Aid in terms of the Standard Vector Deviation,

	Percentage of Observations					
	25	50	75	90	95	99
Radius of Circle	0.23	0.47	0.81	1.17	1.39	1.82

Calculation of Wind Aid

3. Wind aid can also be determined by use of the following formula:

$$W = (A^2 - R^2\sin^2\Theta)^{1/2} + R\cos\Theta - A \qquad (2)$$

where

- W is the wind aid, positive or negative,
- A is the aircraft speed.
- R is the wind speed, and
- Θ is the angle between the wind and the flight path.

The use of the vector mean wind will provide the mean wind aid for any particular aircraft speed and flight path. Table 2 gives the sine and cosine values from 0 to 180° for use in the above formula. Now, to the mean wind aid thus obtained, Table 3 may be used to obtain values which may be added algebraically to determine the percent of occurrence of wind aid equal to or less than a specified value.

Table 2. Sine and Cosine values, 0° to 180°

ϴ	Sin ϴ	Cos ϴ
0°	+.00000	+1.0000
1°	+.01745	+.99985
2°	+.03490	+.99939
3°	+.05234	+.99863
4°	+.06976	+.99756
5°	+.08716	+.99619
6°	+.10453	+.99452
7°	+.12187	+.99255
8°	+.13917	+.99027
9°	+.15643	+.98769
10°	+.17365	+.98481
11°	+.19081	+.98163
12°	+.20791	+.97815
13°	+.22495	+.97437
14°	+.24192	+.97030
15°	+.25882	+.96593
16°	+.27564	+.96126
17°	+.29237	+.95630
18°	+.30902	+.95106
19°	+.32557	+.94552
20°	+.34202	+.93969
21°	+.35837	+.93358
22°	+.37461	+.92718
23°	+.39073	+.92050
24°	+.40674	+.91355
25°	+.42262	+.90631
26°	+.43837	+.89879
27°	+.45399	+.89101
28°	+.46947	+.88295
29°	+.48481	+.87462
30°	+.50000	+.86603
31°	+.51504	+.85717
32°	+.52992	+.84805
33°	+.54464	+.83867
34°	+.55919	+.82904
35°	+.57358	+.81915
36°	+.58779	+.80902
37°	+.60182	+.79864
38°	+.61566	+.78801
39°	+.62932	+.77715
40°	+.64279	+.76604
41°	+.65506	+.75471
42°	+.66913	+.74314
43°	+.68200	+.73135
44°	+.69466	+.71934
45°	+.70711	+.70711
46°	+.71934	+.69466
47°	+.73135	+.68200
48°	+.74314	+.66913
49°	+.75471	+.65606
50°	+.76604	+.64279
51°	+.77715	+.62932
52°	+.78801	+.61566
53°	+.79864	+.60182
54°	+.80902	+.58779
55°	+.81915	+.57358
56°	+.82904	+.55919
57°	+.83867	+.54464
58°	+.84805	+.52992
59°	+.85717	+.51504
60°	+.86603	+.50000

Θ	Sin Θ	Cos Θ
61°	+.87462	+.48481
62°	+.88295	+.46947
63°	+.89101	+.45399
64°	+.89879	+.43837
65°	+.90631	+.42262
66°	+.91355	+.40674
67°	+.92050	+.39073
68°	+.92718	+.37461
69°	+.93358	+.35837
70°	+.93969	+.34202
71°	+.94552	+.32557
72°	+.95106	+.30902
73°	+.95630	+.29237
74°	+.96126	+.27564
75°	+.96593	+.25882
76°	+.97030	+.24192
77°	+.97437	+.22495
78°	+.97815	+.20791
79°	+.98163	+.19081
80°	+.98481	+.17365
81°	+.98769	+.15643
82°	+.99027	+.13917
83°	+.99255	+.12187
84°	+.99452	+.10453
85°	+.99619	+.08716
86°	+.99756	+.06976
87°	+.99863	+.05234
88°	+.99939	+.03490
89°	+.99985	+.01745
90°	+1.0000	.00000
91°	+.99985	-.01745
92°	+.99939	-.03490
93°	+.99863	-.05234
94°	+.99756	-.06976
95°	+.99619	-.08716
96°	+.99452	-.10453
97°	+.99255	-.12187
98°	+.99027	-.13917
99°	+.98769	-.15643
100°	+.98481	-.17365
101°	+.98163	-.19081
102°	+.97815	-.20791
103°	+.97437	-.22495
104°	+.97030	-.24192
105°	+.96593	-.25882
106°	+.96126	-.27564
107°	+.95630	-.29237
108°	+.95106	-.30902
109°	+.94552	-.32557
110°	+.93969	-.34202
111°	+.93358	-.35837
112°	+.92718	-.37461
113°	+.92050	-.39073
114°	+.91355	-.40674
115°	+.90631	-.42262
116°	+.89879	-.43837
117°	+.89101	-.45399
118°	+.88295	-.46947
119°	+.87462	-.48481
120°	+.86603	-.50000
121°	+.85717	-.51504
122°	+.84805	-.52992
123°	+.83867	-.54464
124°	+.82904	-.55919

θ	Sin θ	Cos θ
125°	+.81915	-.57358
126°	+.80902	-.58779
127°	+.79864	-.60182
128°	+.78801	-.61566
129°	+.77715	-.62932
130°	+.76604	-.64279
131°	+.75471	-.65606
132°	+.74314	-.66913
133°	+.73135	-.68200
134°	+.71934	-.69466
135°	+.70711	-.70711
136°	+.69466	-.71934
137°	+.68200	-.73135
138°	+.66913	-.74314
139°	+.65606	-.75471
140°	+.64279	-.76604
141°	+.62932	-.77715
142°	+.61566	-.78801
143°	+.60182	-.79864
144°	+.58779	-.80902
145°	+.57358	-.81915
146°	+.55919	-.82904
147°	+.54464	-.83867
148°	+.52992	-.84805
149°	+.51504	-.85717
150°	+.50000	-.86603
151°	+.48481	-.87462
152°	+.46947	-.88295
153°	+.45399	-.89101
154°	+.43837	-.89879
155°	+.42262	-.90631
156°	+.40674	-.91355
157°	+.39073	-.92050
158°	+.37461	-.92718
159°	+.35837	-.93358
160°	+.34202	-.93969
161°	+.32557	-.94552
162°	+.30902	-.95106
163°	+.29237	-.95630
164°	+.27564	-.96126
165°	+.25882	-.96593
166°	+.24192	-.97030
167°	+.22495	-.97437
168°	+.20791	-.97815
169°	+.19081	-.98163
170°	+.17365	-.98481
171°	+.15643	-.98769
172°	+.13917	-.99027
173°	+.12187	-.99255
174°	+.10453	-.99452
175°	+.08716	-.99619
176°	+.06976	-.99756
177°	+.05234	-.99863
178°	+.03490	-.99939
179°	+.01745	-.99985
180°	+.00000	-1.0000

Table 3. Percentage of wind aid to be determined by adding algebraically these values to the mean wind aid. Wind aid will be equal to or less than the values obtained.

	5%	10%	25%	50%	75%	90%	95%
0	-0	-0	-0	0	+0	+0	+0
1	-1	-1	-0	0	+0	+1	+1
2	-2	-2	-1	0	+1	+2	+2
3	-3	-3	-2	0	+2	+3	+3
4	-5	-4	-2	0	+2	+4	+5
5	-6	-5	-2	0	+2	+5	+6
6	-7	-6	-3	0	+3	+6	+7
7	-8	-7	-3	0	+3	+7	+8
8	-9	-7	-4	0	+4	+7	+9
9	-10	-8	-4	0	+4	+8	+10
10	-12	-9	-5	0	+5	+9	+12
11	-13	-10	-5	0	+5	+10	+13
12	-14	-11	-6	0	+6	+11	+14
13	-15	-12	-6	0	+6	+12	+15
14	-16	-13	-7	0	+7	+13	+16
15	-17	-14	-7	0	+7	+14	+17
16	-19	-14	-8	0	+8	+14	+19
17	-20	-15	-8	0	+8	+15	+20
18	-21	-16	-8	0	+8	+16	+21
19	-22	-17	-9	0	+9	+17	+22
20	-23	-18	-9	0	+9	+18	+23
21	-24	-19	-10	0	+10	+19	+24
22	-26	-20	-10	0	+10	+20	+26
23	-27	-27	-11	0	+11	+27	+27
24	-28	-22	-11	0	+11	+22	+28
25	-29	-23	-12	0	+12	+23	+29
26	-30	-24	-12	0	+12	+24	+30
27	-31	-24	-13	0	+13	+24	+31
28	-33	-25	-13	0	+13	+25	+33
29	-34	-26	-14	0	+14	+26	+34
30	-35	-27	-14	0	+14	+27	+35
31	-36	-28	-15	0	+15	+28	+36
32	-37	-29	-15	0	+15	+29	+37
33	-38	-30	-16	0	+16	+30	+38
34	-40	-31	-16	0	+16	+31	+40
35	-41	-32	-16	0	+16	+32	+41
36	-42	-33	-17	0	+17	+33	+42
37	-43	-33	-17	0	+17	+33	+43
38	-44	-34	-18	0	+18	+34	+44
39	-45	-35	-18	0	+18	+35	+45
40	-47	-36	-19	0	+19	+36	+47
41	-48	-37	-19	0	+19	+37	+48
42	-49	-38	-20	0	+20	+38	+49
43	-50	-39	-20	0	+20	+39	+50
44	-51	-40	-21	0	+21	+40	+51
45	-53	-40	-21	0	+21	+40	+53
46	-54	-41	-22	0	+22	+41	+54
47	-55	-42	-22	0	+22	+42	+55
48	-56	-43	-23	0	+23	+43	+56
49	-57	-44	-23	0	+23	+44	+57
50	-58	-45	-24	0	+24	+45	+58
51	-60	-46	-24	0	+24	+46	+60
52	-61	-47	-24	0	+24	+47	+61
53	-62	-48	-25	0	+25	+48	+62
54	-63	-49	-25	0	+25	+49	+63
55	-64	-50	-26	0	+26	+50	+64
56	-66	-50	-26	0	+26	+50	+66

	5%	10%	25%	50%	75%	90%	95%
57	-67	-51	-27	0	+27	+51	+67
58	-68	-52	-27	0	+27	+52	+68
59	-69	-53	-28	0	+28	+53	+69
60	-70	-54	-28	0	+28	+54	+70
61	-71	-55	-29	0	+29	+55	+71
62	-73	-56	-29	0	+29	+56	+73
63	-74	-57	-30	0	+30	+57	+74
64	-75	-58	-30	0	+30	+58	+75
65	-76	-58	-31	0	+31	+58	+76
66	-77	-59	-31	0	+31	+59	+77
67	-78	-60	-31	0	+31	+60	+78
68	-80	-61	-32	0	+32	+61	+80
69	-81	-62	-32	0	+32	+62	+81
70	-82	-63	-33	0	+33	+63	+82
71	-83	-64	-33	0	+33	+64	+83
72	-84	-65	-34	0	+34	+65	+84
73	-85	-66	-34	0	+34	+66	+85
74	-87	-67	-35	0	+35	+67	+87
75	-88	-68	-35	0	+35	+68	+88
76	-89	-68	-36	0	+36	+68	+89
77	-90	-69	-36	0	+36	+69	+90
78	-91	-70	-37	0	+37	+70	+91
79	-92	-71	-37	0	+37	+71	+92
80	-94	-72	-38	0	+38	+72	+94
81	-95	-73	-38	0	+38	+73	+95
82	-96	-74	-39	0	+39	+74	+96
83	-97	-75	-39	0	+39	+75	+97
84	-98	-76	-39	0	+39	+76	+98
85	-99	-76	-40	0	+40	+76	+99
86	-101	-77	-40	0	+40	+77	+101
87	-102	-78	-41	0	+41	+78	+102
88	-103	-79	-41	0	+41	+79	+103
89	-104	-80	-42	0	+42	+80	+104
90	-105	-81	-42	0	+42	+81	+105
91	-106	-82	-43	0	+43	+82	+106
92	-107	-83	-43	0	+43	+83	+107
93	-108	-84	-44	0	+44	+84	+108
94	-110	-85	-44	0	+44	+85	+110
95	-111	-86	-45	0	+45	+86	+111
96	-112	-86	-45	0	+45	+86	+111
97	-113	-87	-46	0	+46	+87	+113
98	-114	-88	-46	0	+46	+88	+114
99	-116	-89	-47	0	+47	+89	+116
100	-117	-90	-47	0	+47	+90	+117

4. Example

At a point along a prescribed route the vector mean wind is 315° and 100 knots while the standard vector deviation is 50 knots. The aircraft velocity is 090° and 300 knots.

A. $W = (A^2 - R^2 \sin^2 \Theta)^{1/2} + R\cos\Theta - A$

$W = (90,000 - 10,000 \cdot 0.5)^{1/2} + 100 \cdot 0.707 - 300$

$W = 291.5 + 70.7 - 300 = 62.2$

The mean wind aid is 62.2 knots.

B. From Table 3 the following values are obtained and added to 62.2 knots.

5%	10%	25%	50%	75%	90%	95%
-58.0	-45.0	-24.0	0.0	+24.0	+45.0	+58.0
62.2	62.2	62.2	62.2	62.2	62.2	62.2
+ 4.2	+17.2	+38.2	+62.2	+86.2	+107.2	+120.2

Thus 10% of the time, wind aid is 17.2 knots or less

75% of the time, wind aid is 86.2 knots or less

95% of the time, wind aid is 120.2 knots or less.

The last can be interpreted also as 5% of the time the wind aid is greater than 120.2 knots.

5. Limitations: It must be remembered that these tables can only be used where the distribution of winds is circular. Where the distribution is elliptical, other methods have to be applied, the easiest of which is graphical.

6. Application of Method to Route Wind Aid.

A correlation between point winds and route winds exists. Table 4 which is modified from Sawyer's paper gives the correction factor which must be applied to the average standard vector deviation over the route before the route equivalent wind can be obtained.

Table 4. Multiplier factor to convert the Mean Standard Vector Deviation of wind over a route into the standard deviation of wind aid.

Length of route (nautical miles)	0	500	1000	1500	2000	2500	3000	3500	4000
Factor	0.71	0.66	0.60	0.54	0.49	0.45	0.43	0.39	0.37

Table 5 gives the required data for computing wind aid along the Oakland, Cal. - Rantoul, Illinois route. The data is for the 300 mb. pressure surface during the winter season, Dec. - Jan. - Feb., 1948 - 1953. $\overline{V}$ is the vector mean wind and σ_v is the standard vector deviation.

Table 5. Wind Data for the Oakland, Cal. - Rantoul, Ill. route at the 300 mb., Dec. - Jan. - Feb., 1948 - 1953.

Station	$\overline{V}$ Direction (degrees)	Speed (knots)	σ_v
Oakland, Cal.	290°	40	48
Ogden, Utah	279°	42	48
Denver, Colo.	274°	47	48
Omaha, Nebr.	270°	58	49
Rantoul, Ill.	268°	66	49

Step 1. An average flight path for the Oakland - Rantoul route would be towards 073° while the vector mean wind for the route is from 275° and 49 knots. The mean equivalent wind is then computed by the following formula:

$$W = (A^2 - R^2 \sin^2\Theta)^{1/2} + R\cos\Theta - A \qquad (1)$$

where A is the aircraft speed, R is the magnitude of vector mean wind and Θ is the angle between the flight path and the vector mean wind. Here A = 300, R = 49 and $\Theta = 22°$. W = 45 knots.

Step 2. In order to obtain the distribution of equivalent winds about this mean, it is only necessary to average the standard vector deviations and apply a correction factor from table 4.

The average of the deviation is 48.4 while the route distance is about 2,000 nautical miles. The standard deviation of the wind components along the route is the 48.4 x .49 is 24 knots.

Step 3. On arithmetic probability paper plot the mean wind aid of 45 knots at the 50th percentile. At the 84th percentile plot (45 + 24) knots or 69 knots. Such a procedure is illustrated in Fig. 1 by the solid line. Also plotted for comparison is the route wind aid published by the U. S. Navy. The agreement is considered to be excellent for rounding to the nearest whole percent causes a loss of 2 percent in summation of the published value. An addition of 1% at the 95% point and above the 99% point will decrease the small difference shown.

7. Conclusion.

Point and Route wind aid may be computed by Sawyer's methods which utilize the vector mean wind and the standard vector deviation. These methods can only be utilized if the wind distributions are distributed in the circular normal sense. If the distributions are elliptical, then graphical procedures, as presented in a paper before the A. M. S. at Miami, Fla. (1954), can be more easily utilized.

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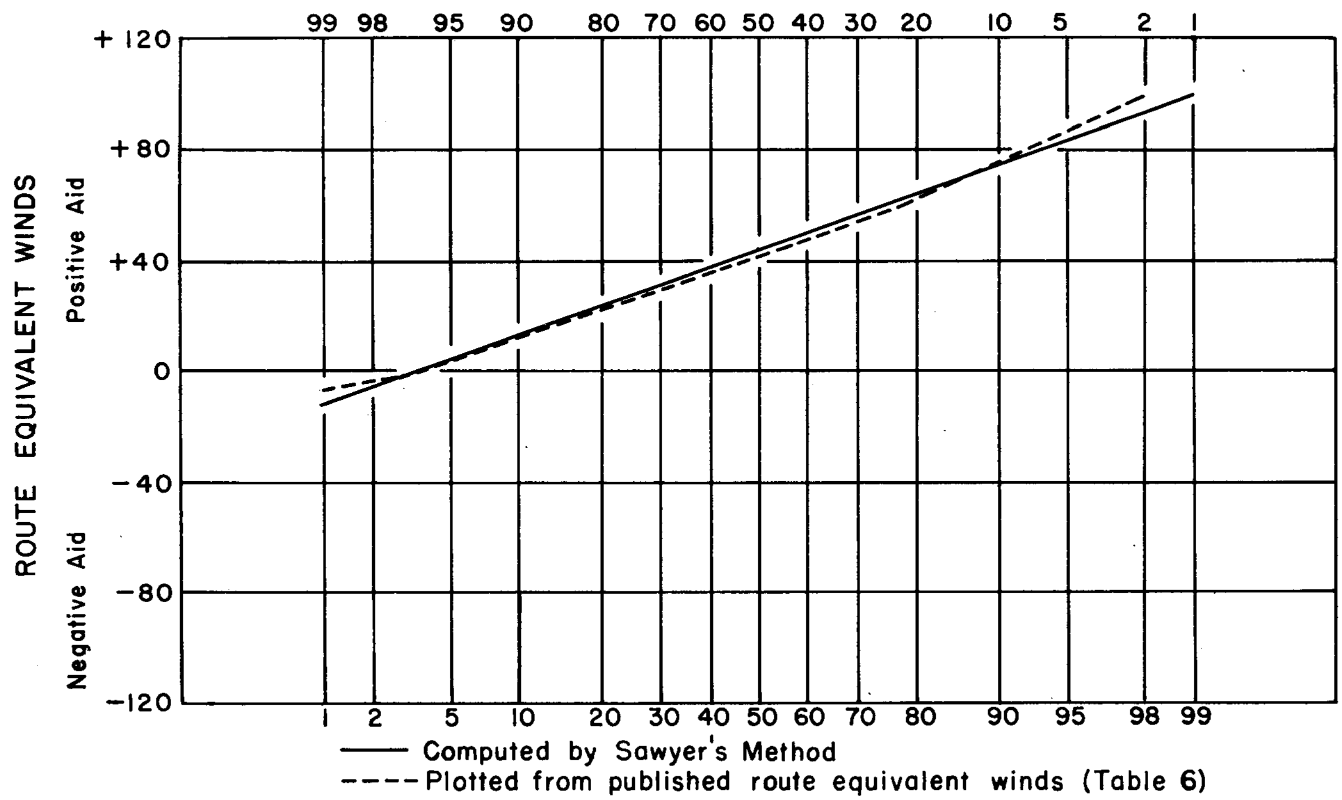

ROUTE EQUIVALENT WINDS
ON THE OAKLAND, CAL. – RANTOUL, ILL. ROUTE
300 Millibars; 300 Knot Aircraft Speed; Dec.- Jan.-Feb.

Fig. 1