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ON THE STANDARD VECTOR-DEVIATION WIND ROSE

By

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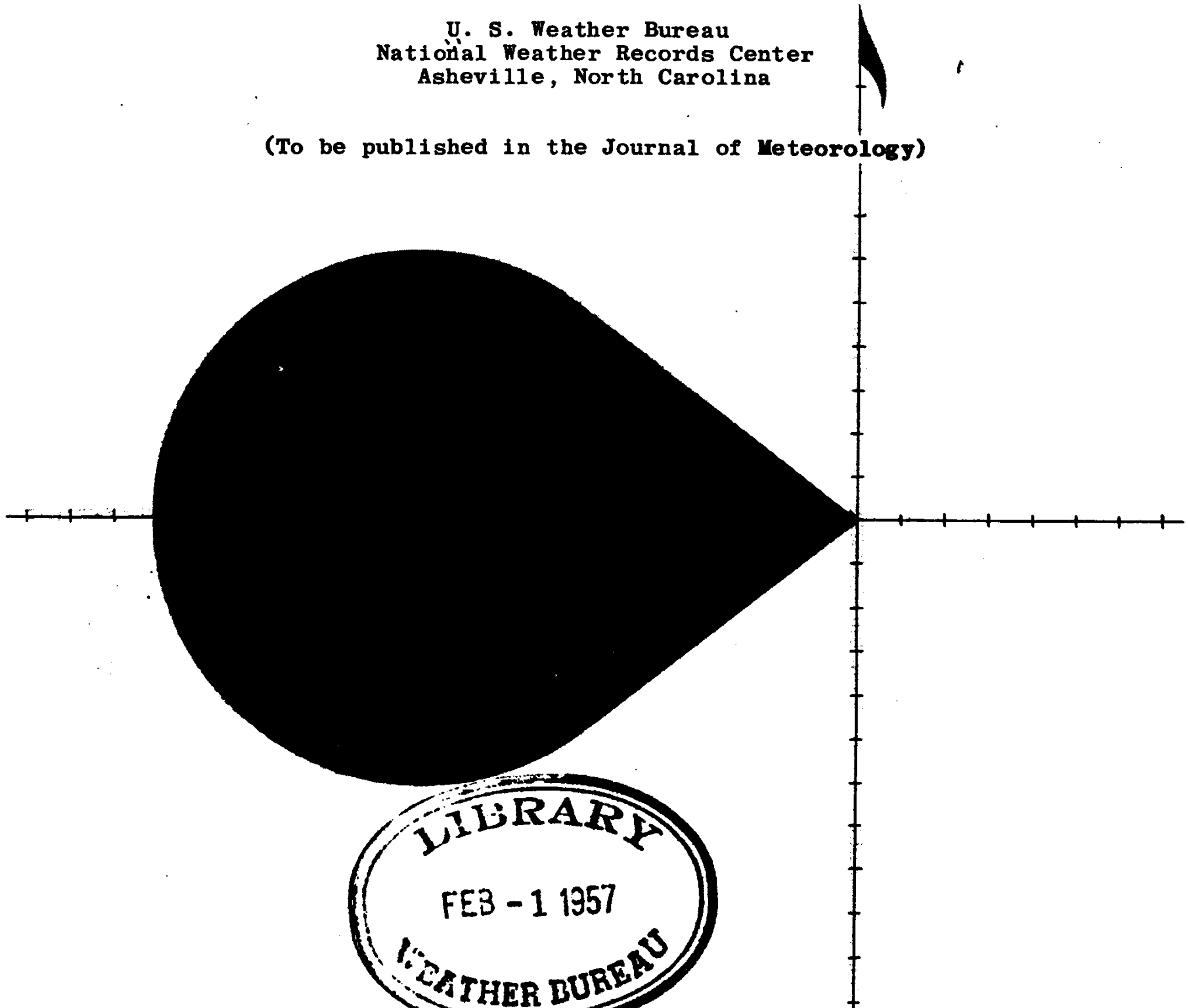

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ABSTRACT

Wind data presentations are varied. Some serve one purpose better than another. A relatively new type of wind rose is presented in order that its potential may be exploited. This is the Standard Vector-Deviation Wind Rose. It is an easily reproduced rose and may be constructed from values of the vector mean wind, standard deviations of the latitudinal and meridional wind components and the correlation between the components. If the distribution is circular, rather than elliptical, only the vector mean and the standard vector deviation are needed. Methods to test for ellipticity and to construct the roses are given.

Explanation of the rose on the cover.

Tokyo, Japan	1948-1953	200 mbs	Spring, March-April-May
Vector Mean Wind	270°	100 mph	
standard vector deviation		50 mph	

The speed increments are ten miles per hour and the circles enclose 25, 50 and 75 percent of the winds.

1. Introduction

From time immemorial, man has attempted to depict the nature of his environment and its unique characteristics which affected him most strongly. Weather and climate often created situations which caused him untold hardships. Wind was a phenomenon which to him accompanied his "present weather" and was a harbinger of the future. As winds sometimes were beneficial and at other times harmful, it was inevitable that certain winds were given special names; e.g., the bora, the mistral, and the chinook. To depict wind information, various methods of representations have been developed. Each one has its advantages and its disadvantages. Some of these representations are illustrated in Fig. 1.

2. Purpose

It is the purpose of this paper to help present a relatively new type wind rose which is termed here, a Standard Vector-Deviation Wind Rose, to be referred to hereafter as the SVD Wind Rose. The development of this rose has been slow and its origin perhaps can be traced to the vector error distribution in measurements employed in the physical sciences. Its development in the meteorological field is given in a subsequent section of this paper.

3. General Discussion of Wind Roses

To understand a difficulty into which the conventional wind roses tend to lead one, let us consider the breakdown of distribution percentage from the 4-point to the 8-point and then to the 16- and 32-point roses. As the breakdown continues, the percentage which falls to each point and speed interval becomes less and less until it seems absurd to proceed further. At some point the user is forced to retrace his steps until he obtains some concept of the distribution. In any case, when he talks about a certain part of the distribution, he talks in an areal sense rather than a point sense.

To some users, the 8-point rose is sufficient for their needs for they have learned to interpret this rose in so far as they are personally concerned. To another user, yet for the same purpose, a 16-point rose is absolutely necessary. This implies that each rose must be correlated with a particular problem by the user.

4. Theory and the SVD Wind Rose

A. The Theory of the Bivariate Distribution

Bravais 1 in 1846 published his remarkable study of normal frequency distributions in two and more than two variables. This study, initiated a decade earlier, was based on the work on the univariate distribution and probabilities by earlier well-known mathematicians. In 1888, Bertrand [2] developed the theory still further and gave the equations for vector distributions. The first examples of such distribution studies seems to be the well-known placement of rifle shots on a target where the distance from the center to a particular shot may be considered to be a vector equal to the horizontal and vertical deviations of that shot. In practice, it is noted that these shots seem to be correlated. Cotes, according to Bertrand, first proposed the assumption that if any number of shots have struck the target, then the most probable position of the point aimed at is the centroid of these points. Then, too, Fermat's well-known Principle of Conjunctive Probability considers the probability that two events will happen. This probability is hk where h is the probability that the first event will happen and k is the probability that the second event will happen when the first is known to have happened.

Eq. 1 modified after Bertrand describes the bivariate distribution. According to Hald [3] , this may also be written as a sum of squares of two stochastically independent variables. This quantity is distributed as chi-squared with two degrees of freedom.

$$\ln_e 1/(1 - p) = \frac{1}{2(1-\rho^2)} \left[\frac{(X-\bar{X})^2}{\sigma_x^2} + \frac{(Y-\bar{Y})^2}{\sigma_y^2} - 2\rho \frac{(X-\bar{X})}{\sigma_x}\frac{(Y-\bar{Y})}{\sigma_y} \right] \quad (1)$$

As written here, the respective variances of the variates x and y are σ_x^2 and σ_y^2 while the correlation between them is ρ. The standard deviations are σ_x and σ_y. For a selected percentage, P, the left-hand member is a constant. The equation, then, represents an ellipse in the (X,Y) plane with the center at $(\bar{X},\bar{Y})$ and axes that are parallel to the coordinate axes only if ρ is not different from zero. The bar above a symbol indicates an averaging process.

To determine if ρ is significantly different from zero, the "t" distribution may be used as shown in Eq. 2. Here, "f" equals n-2 and "n" is the number of observations. If the value of "t" is less than the 95th percentile value extracted from standard statistical tables, a conclusion that the two variates are correlated is not justified. Therefore, in subsequent calculations, ρ may be assumed to be zero.

$$t = \left[\rho/(1 - \rho^2)^{1/2}\right] f^{1/2} \tag{2}$$

If, however, it is concluded that correlation exists, the angle of rotation, ψ, between the coordinate and ellipses' axes may be determined by Eq. 3. If ρ is positive, the major axes slope upwards to the right.

$$\tan 2\psi = 2\rho\sigma_x \sigma_y/(\sigma_x^2 - \sigma_y^2) \tag{3}$$

The variances along the major and minor axes will not be, in this case, the same as those along the coordinate axes. According to Mauchly [4], Eq. 4 may be used to compute the new variances. In this equation, $k_1 > k_2$ and both are invariant under rotation of axes. Here, $(k_1)^{1/2}$ and $(k_2)^{1/2}$ are the standard deviations along the major and minor axes and will be denoted as σ_a and σ_b, respectively.

$$\phi(k) = \begin{vmatrix} \sigma_x^2 - k & \sigma_x \sigma_y \rho \\ \sigma_x \sigma_y \rho & \sigma_y^2 - k \end{vmatrix} = 0 \tag{4}$$

In order to determine the ellipticity of a distribution, Mauchly defines an ellipticity statistic, L_e, as in Eq. 5.

$$L_e = 2\sigma_x \sigma_y (1 - \rho^2)^{1/2} \Big/ \left(\sigma_x^2 + \sigma_y^2\right) \tag{5}$$

If $\sigma_x = \sigma_y = \sigma_a = \sigma_b$ and $\rho = 0$, $L_e = 1$; otherwise, L_e is less than one. The probability of obtaining a value as small as L_e in a sample of "n" independent observations drawn from a population in which L_e equals 1, is shown by Mauchly to be $L_e^{\,n-2}$. Whittaker and Robinson [5] show that the standard vector-deviation, which they denote as σ_v and which is denoted here as $\sigma_{\underline{v}}$ is given by Eq. 6.

$$\sigma_{\underline{v}} = \left[\sigma_x^2 + \sigma_y^2 - 2\rho\sigma_x\sigma_y\right]^{1/2} \qquad (6)$$

If $L_e = 1$, then $\sigma_{\underline{v}} = \left[\sigma_x^2 + \sigma_y^2\right]^{1/2} = \left[\sigma_a^2 + \sigma_b^2\right]^{1/2}$, and the circular distribution is attained as the special limiting case of an elliptical distribution. Further information on the bivariate and multivariate distributions may be obtained by reference to Kendall [6]. Bartels [7], in geomagnetic studies, and Chapman [8], in atmospheric lunar tides, have utilized the bivariate distribution. Hotelling [9] applied these principles in establishing quality control of bomb production.

B. Application of Theory to Wind Distributions

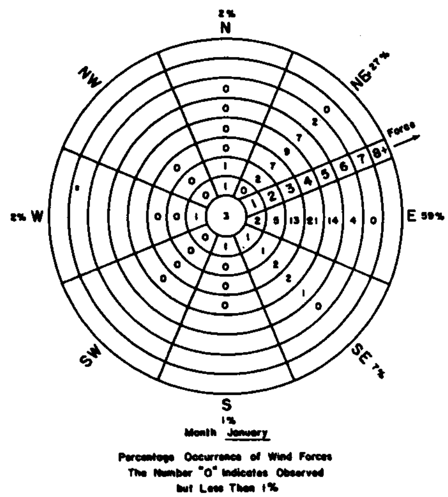
Robitzsch [10] in his 1917 study on the structure of surface wind speeds stated that the wind distribution might follow the probability law proposed by Maxwell. Hesselberg and Bjorkdal in 1929 [11], following this line of reasoning, published their paper on the distribution of wind turbulence. In this paper, they presented the equation for wind distributions. Brooks and collaborators [12] followed with their important treatises on the upper winds over the world, wherein the limiting elliptical distribution, the circular, is employed.

Under the assumption of normal distribution, Brooks states that it should be possible to represent any wind rose by only two parameters: the vector mean wind and the standard vector-deviation; and that from these two parameters, any conventional wind rose can be developed. The standard vector-deviation is a standard length taken as a measure of dispersion in all directions about a vector mean. It is analogous to the standard deviation in its relation to the scalar mean. It may be represented by a circle centered at the head of the vector

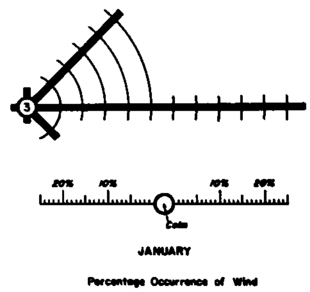
OCEAN AREA MARSDEN SQUARE
10°−044 5°−50
Lat. 010 − 020N
Long. 070−080W
664 Observations

OCEAN AREA 10°-044	MARSDEN SQUARE 5°-50									Month JANUARY
Latitude 010-020 N Longitude 070-080 W 664 observations.										The numbers "0" indicates observed but less than 1%
PERCENTAGE OF OCCURRENCE OF WIND FORCES										
Beaufort Force	0	1	2	3	4	5	6	7	8	Total
Direction										
Calm	3									
N		1	1	0	0	0	0			2
NE		0	2	7	9	7	2	0		27
E		2	5	13	21	14	4	0		59
SE		1	1	2	2	1	0			7
S		1	0	0	0					1
SW		0	0	0	0					0
W		1	0	0						2
NW		0	0	0						0

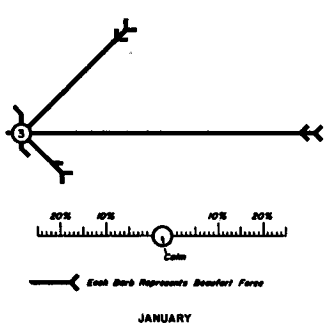
a. Contingency Table



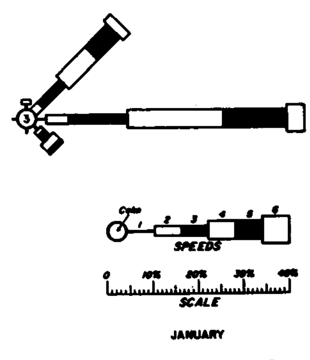
b. Contingency Wind Rose



c. Wind Rose (direction only)



d. Wind Rose (in Beaufort Forces)



e. Wind Rose (Baillie Type)

Fig. I Illustrations of wind distribution for Marsden Square Area, Lat. 010−020N, Long. 070−080W. 664 observations.

mean with a radius equal to its magnitude. <u>But why not let the wind rose implied in this argument stand by itself, on its own merits</u>. Where it fails to answer some questions, then recourse may be had to its modification to another type of wind rose as Brooks has indicated.

Nearly everyone is familiar with the presentation of the resultant wind on charts. To those who have examined these charts, the question always arises: "Just how much does the wind vary around this resultant; do the directions oscillate very much and/or are the speeds relatively constant". These particular questions have been answered, in part, by the formulation and presentation of a term which is usually called the "constancy". The SVD Wind Rose also serves to depict the wind and its variation.

Circular Distributions

Fig. 2a, modified from Hesselberg and Bjorkdal's paper, illustrates the vector mean wind and the standard vector-deviation. Here, the vector mean wind is denoted by $\bar{\underline{V}}$, the standard vector deviation by $\sigma_{\underline{V}}$. An individual wind is

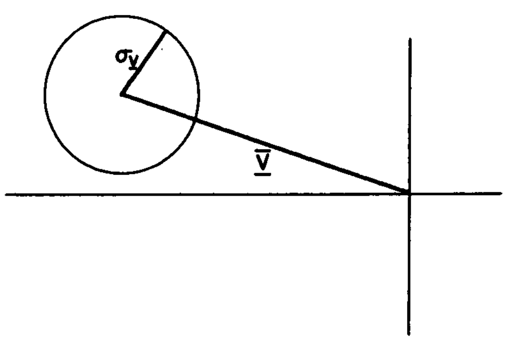


Fig. 2a

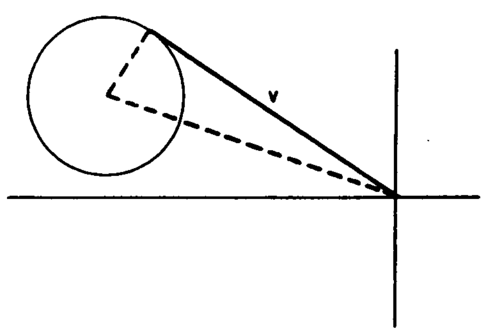


Fig. 2b

Fig. 2. Schematic illustration of wind components and parameters.

(a) Resultant wind (vector mean wind), $\bar{\underline{V}}$, and standard vector deviation, $\sigma_{\underline{V}}$, (modified from Hesselberg and Bjorkdal [11]).

(b) Extension of (a) to show the relation of an individual observation to $\bar{\underline{V}}$ and $\sigma_{\underline{V}}$.

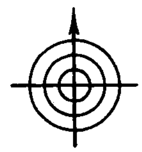
denoted by v in Fig. 2b. The vector mean (resultant) wind is computed by well-known methods. The standard vector deviation is computed by means of Eq. 7.

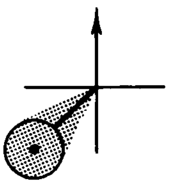
$$\sigma_{\underline{v}} = \left[\Sigma v^2/n - \bar{\underline{V}}^2\right]^{1/2} \tag{7}$$

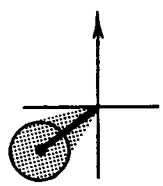
Here Σv^2 is the sum of the squares of the individual speeds and $\bar{\underline{V}}^2$ is the square of the vector mean wind magnitude. The circle described in Fig. 2a with a radius equal to $\sigma_{\underline{v}}$ contains the origins of 63 percent of the distributed winds, if the distribution is normal and circular. The origin of the axes is considered to be the head of all vectors. To obtain this particular radius and the radii of other circles encompassing various portions of the distributed winds, Eq. 1 may be used. In the circular distribution, where $\rho = 0$, the denominator of Eq. 1 reduces to $2\sigma_x^2$. From Eq. 7, this then is seen to be equal to $\sigma_{\underline{v}}^2$. Once the magnitude of $\sigma_{\underline{v}}$ has been determined, any distribution circle may be drawn. The radius of a particular circle will be the product of $\sigma_{\underline{v}}$ and the value obtained from Eq. 1 or from Table 1 which is an amplification of Bertrand's tables. For example, a radius of $0.83\sigma_{\underline{v}}$ will describe a circle which encloses 50 percent of the winds. Such a circle is illustrated schematically in Fig. 3.

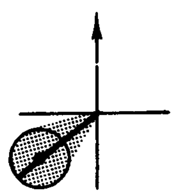
Fig. 3. Schematic illustration as to how to extract information from graphical presentation of circular standard vector deviation wind roses. A fifty percent distribution is shown. This procedure is applicable to elliptical distributions.

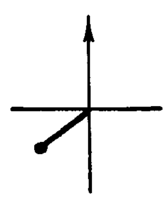
1.  North

2.  Speeds, radii increments are 20 knots each.

3.  Minimum Wind Speeds. The magnitude of the minimum wind speed for the central 50% distribution is shown by the smallest distance from the compass center to the 50% distribution circle.

4.  Vector Mean Wind Speeds. The magnitude of the vector mean wind speed is shown by the distance from the large dot to the intersection of the axes.

5.  Maximum Wind Speeds. The magnitude of the maximum wind speed for the central 50% distribution is shown by the greatest distance from the compass center to the 50% distribution circle.

6.  Direction of vector mean wind or resultant mean wind can be indicated by line extended from the large dot to the intersection of the axes.

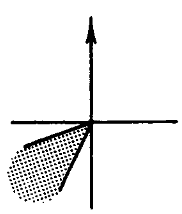
7.  More than 50% of the time winds will be observed to have a direction from that portion of the compass indicated by the shaded sector area.

Fig. 3

Table 1. Values of $[\ln_e 1/(1-P)]^{1/2}$ for values of P ranging from 5 to 99 percent.

P	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$[\ln_e 1/(1-P)]^{1/2}$	0.22	0.32	0.40	0.48	0.54	0.59	0.66	0.72	0.78	0.83
P	0.55	0.60	0.63	0.70	0.75	0.80	0.85	0.90	0.95	0.99
$[\ln_e 1/(1-P)]^{1/2}$	0.89	0.96	1.00	1.10	1.18	1.27	1.38	1.52	1.73	2.15

Fig. 4 shows an SVD Wind Rose for Charleston, South Carolina, at the 300 millibar surface for the months of December, January and February. Distribution circles are drawn for the 25, 50 and 75 percent distributions. A more comprehensive concept of the wind distribution is obtained.

For comparison, a contingency wind rose is superimposed on the SVD Wind Rose in Fig. 4. This gives the percentages to eight points of the compass as extracted from the publication by the U. S. Navy [13] . It will be seen that the count varies little from the distribution indicated by the theoretical SVD Wind Rose. For example, the count shows 73 within the 75 percent circle. This difference is negligible when it is considered that the class intervals used were 20 knots.

Elliptical Distributions

If the observed winds come from distinctly different regimes as are found in differing air masses, the distributions may not be circular but may be elliptical, or as with the case of surface winds, quite distorted as to pattern. In the latter case, it is doubtful if a single rose would adequately describe the distributions. Other examples will be found at the boundary between the westerly and the trade winds, between the top of a monsoon circulation and that above it, at the circulation above the tropopause and above mountain regions. Also the boundaries between continental and maritime areas may exhibit elliptical patterns. Areas of strong divergence and convergence such as blocking areas and preferred regions of jet stream development will be characterized by such patterns.

A review of 1000 seasonal upper air distributions during 1948-53 over the area extending from Europe westward to Asia indicates that about 1/6 of those over maritime areas and 1/3 of those over continental areas should be considered to be elliptical. In the application of ellipticity tests, where there may be a dependence of one observation of one day to the preceding, the effect of this dependence may be eliminated in part by dividing the number of observations, n, by 2. If the correlation is as high as 0.20, the quantity $(1 - \rho^2)^{1/2} = 0.98$. At the 5 percent level of significance and with 100 observations, $L_e = 0.97$. Substituting this into Eq. 5 and setting $\sigma_y = 1$, the value of σ_x may range from 0.91 to 1.11 of the value of σ_y. As correlations are not generally higher than 0.20, these limits will usually be acceptable. If the ratio of the standard deviations exceeds the limits shown above, the distribution should be considered to be elliptical.

It is now seen that with the parameters $\bar{Y}$ and σ_Y for the circular distributions or with $\bar{Y}$, σ_a, σ_b, and ρ for the elliptical ones, wind roses may be constructed on polar coordinate paper and information extracted as shown in Fig. 3. With reference to the last, it will be noted that the percentage of time that winds come from the sector containing a circle or ellipse will be greater than that contained on or within the circle or ellipse, itself. Just how much more depends, in part, on the vector mean, the scalar mean and the standard vector deviation. With these parameters, tables given by Brooks and Carruthers [14] may be used to obtain the percentage occurrence of wind from within 20° sector areas to either side of the vector mean. The distribution of scalar speeds within each sector can also be determined. However, these are applicable only to the circular distribution.

A qualitative idea as to the magnitude of the percentage occurrence of winds within any particular sector can be obtained without recourse to the above tables. In order to obtain an idea of the former, consider the construction of two distribution ellipses such as the 50 and 70 percent ellipses. The sectors formed by rays drawn from the origin of the coordinate axes tangent to the ellipses (circles) and extended beyond them will contain, respectively, more than 50 percent and more than 70 percent of the winds. A sector bounded by two of the rays, one from each ellipse, will then contain less than 10 percent of the winds. This is caused by the fact that the percentage greater than the 50 percent is larger than the percentage greater than the 70 percent. By judicious selection of ellipses of specific percentage or selection of desired sector areas and subsequent determination of respective ellipses bounded by the sectors, a close approximation to the actual percentages in each sector can be determined. The distribution of speeds within each sector is difficult.

As an example of an elliptical distribution, the winter season at 300 mbs over Omaha, Nebraska, is illustrated. Over a five year period, 1948-1953, December through February, 480 observations were collected. The vector mean wind is 270° and 60.0 knots. The standard deviations along the E-W and N-S axes, σ_x and σ_y, respectively, are 30.6 and 36.8 knots. The correlation between components is 0.23. By means of Eq. 2, the correlation is found to be highly significant and must, therefore, be used. Its computed value is 5.2. Too, the ratio of σ_x to σ_y is 0.80, and the distribution must be considered to be elliptical. By means of Eq. 4, σ_a and σ_b are found to be 38.4 and 28.6 knots, respectively. These, of course, must be multiplied by $2^{1/2}$, which gives 54.4 and 40.3 knots. The magnitudes of the axes of the various ellipses may then be determined by means of Table 1 for the selected percentages of 25, 50 and 75. The angle of rotation, ψ, is 065°. The rose is then constructed by laying off a vector mean magnitude of 60 knots from the west, 270° on polar coordinate paper. The major axes of the ellipses lie along a line which is rotated counterclockwise 065° around the centroid of distribution. Rotation begins from a line parallel to the X or E-W axis and the centroid is located at the origin of the vector mean; i.e., it is 60 knots from the origin of the coordinate axes. The wind rose constructed with the above criteria is shown in Fig. 5. Superimposed on this is a contingency wind rose for comparison as in Fig. 4.

Conclusion

The representation of wind distributions has always been difficult. Witness to this fact is the many and varied forms which such representations have taken. The standard vector-deviation wind rose is a basic representation for it gives at a quick glance a concept of both the magnitudes and the variability of the winds. Wind rose charts of the above type would permit a quick relative areal determination of winds and their variability.

Procedures suggested above will permit a general condensation of much information into a few charts. These charts would include the vector mean wind isotachs, the standard deviations along the distribution axes, and the angle the major axis makes with the east-west direction. In lieu of these standard deviations, those along the latitudinal and meridional axes and the correlation coefficient could be used. The required values to ascertain wind distributions at any point in space could be extracted from these charts. In addition such charts would provide useful information to those engaged in research.

The SVD Wind Rose, although it will not provide ready answers to some questions, which are answered in other wind roses, does provide answers where other wind roses are deficient. It, therefore, deserves a place with the others. It is readily applicable to problems of aeronautical and nautical navigation. Also the principles involved may be used in studying movement of storms or ocean currents just as they have been used in many other fields of the physical sciences.

REFERENCES

1. Bravais, A., 1846: Analyse mathematique sur les probabilities des erreurs de situation d'un point. Mem. presentes par divers savants. Academie des Sciences Mem. Sav. Etrang., Paris, 9, 255-332.
2. Bertrand, J., 1888: Calcul des probabilities: Note sur la probabilite du tir a la cible: Troiseme note sur la probabilite du tir a la cible. Academie des Sciences Comptes Rendus, Paris, 106, 387-391, 521-522.
3. Hald, A., 1952: Statistical Theory with Engineering Applications, New York, John Wiley and Sons, 585-612.
4. Mauchly, J. W., June 1940: A significance test for ellipticity in the harmonic dial. Terrestrial Magnetism and Atmospheric Electricity, 45, No. 2, 145-148.
5. Whittaker, E., and Robinson, G., 1952: The Calculus of Observations, fourth edition, London and Glasgow, Blackie and Son, Lmtd., 321-327.
6. Kendall, M. G. 1952: The Advanced Theory of Statistics, Vol. 1, fifth edition, New York, Hafner Publishing Co., 19-22.
7. Bartels, J., May 1932: Statistical Methods for Research on Diurnal Variations. Terrestrial Magnetism and Atmospheric Electricity, 37, 291-302.
8. Chapman, S., 1951: Atmospheric Tides and Oscillations. Compendium of Meteorology, Edited by T. F. Malone, Boston, Mass., American Meteoro-

logical Society, 510-530.

9. Hotelling, H., 1950: The Generalized T Test. Second Berkeley Symposium on Mathematical Statistics and Probability, Edited by Jerzy Neyman, Univ. of California Press, 23-41.

10. Robitzsch, M., 1917: Beitrage zur Kenntnis der Struktur des Bodenwindes. Die Arbeiten des Preupischen Aeronautischen Observatoriums bei Lindenberg, XIII Band, Wissen, Prb. Braunschweig, 66-94.

11. Hesselberg, T. and Bjorkdal, E., 1929: Uber das Verteilungsgesetz der Windunruhe. Beitrage zur Physik der Freien Atmosphare, Band XV, 121-133.

12. Brooks, C. E. P., Durst, C. S., and Carruthers, N., 1946: Upper Winds Over the World. Part I. The Frequency Distribution of Winds at a Point in the Free Air. Quarterly Journal of the Royal Meteorological Society, 72, No. 311, 55-73.

13. U. S. Navy, January 1954: Tables of Winds and Their Aiding and Retarding Effect. NAVAER 50-1C-526, Aerology Branch, Office of the Chief of Naval Operations, U. S. Navy.

14. Brooks, C. E. P., Durst, C. S., and Carruthers, N., Dewar, D., and Sawyer, J. S., Upper Winds Over the World, M. O. 499e, Air Ministry Meteorologiccal Office, Geophysical Memoirs N. 85 (Fifth Number Volume X) London: His Majesty's Stationery Office 1950.

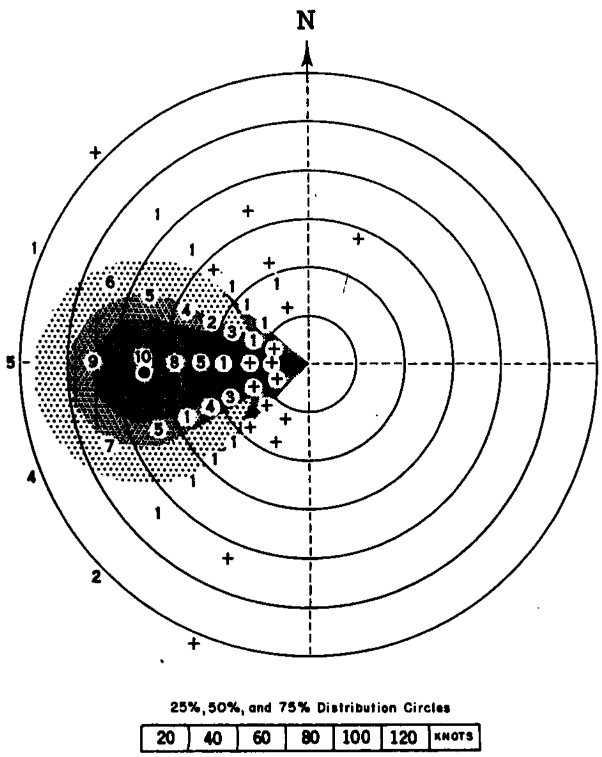


Fig. 4 STANDARD VECTOR DEVIATION WIND ROSE

STATION Charleston, S. C.
PRESSURE LEVEL 300 millibars
SEASON Winter
PERIOD 1948-1953

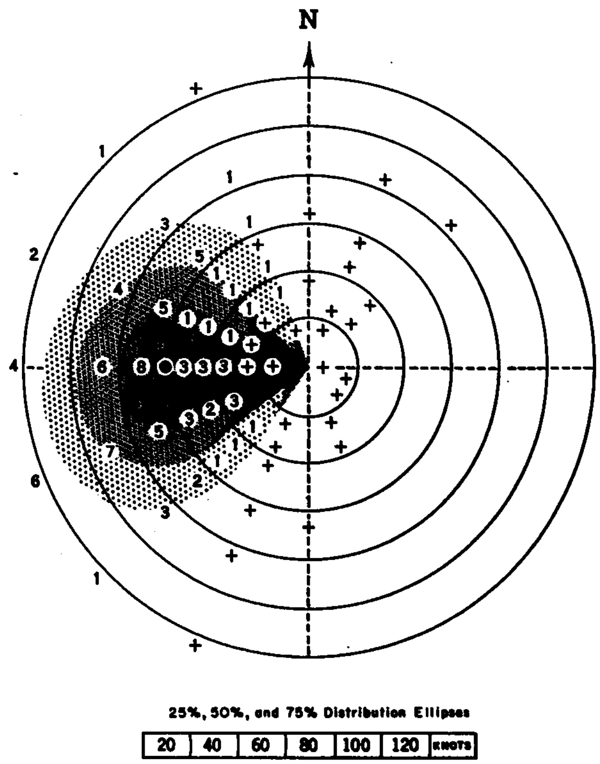


Fig. 5 STANDARD VECTOR DEVIATION WIND ROSE

STATION Omaha, Nebr.
PRESSURE LEVEL 300 millibars
SEASON Winter
PERIOD 1948-1953